Perpetual commodity equities in extended Kalman-Bucy linear filtering models for convenience yields

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Abstract

We study in the problem of pricing of the perpetual commodity equities in an extension of the Black-Scholes-Merton model for commodity markets in which the (unobservable signal process) convenience yield rates are modelled by Ornstein-Uhlenbeck processes. In this case, the asset prices are modelled by exponential time-inhomogeneous diffusion processes which play the role of the observations in the extended two-dimensional Kalman-Bucy linear filtering problem with either independent or constantly correlated driving standard Brownian motions. The commodity equity pricing problems are formulated as either a (discounted) time-homgeneous optimal stopping problem for a two-dimensional continuous (Markov) diffusion process (full information) or a time-inhomogeneous optimal stopping problem for a one-dimensional diffusion process (partial information). The rational valuation of contracts of such type on the infnite time horizon and with the default opportunities in the classical model based on a one-dimensional geometric Brownian motion with constant coeffcients was studied in Leland [2]. In [1] the necessarily two-dimensional optimal stopping problem was studied which is associated with the one of the pricing of commodity equities with the commodity spot price expressed by a generalised geometric Brownian motion having the drift rate described by the filtering estimate of an unobservable continuous-time Markov chain with two states (Wonham filter).

It is shown that the optimal times of exercise is the first time at which the commodity spot price paid in return to the fixed coupon rate hits a lower stochastic boundary being a monotone function of the running value of either the filtering estimate (partial information) or the signal process (full information). We rigorously prove that the optimal stopping boundary is regular for the stopping region relative to the resulting two-dimensional diffusion process and the value function is continuously differentiable with respect to the both variables. The optimal stopping problems are reduced to the equivalent parabolic-type (partial information) or elliptic-type (full information) free-boundary problems. It is verified by means of the change-of-variable formula with local time on surfaces from [3, Theorem 3.1] (cf. also [4, Chapter II, Section 3.5] for a summary of the related results and further references) that the value functions and the boundaries are determined as unique solutions of the associated nonlinear Fredholm-type integral equations.

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References

[1] Gapeev P. V. (2022), "Discounted optimal stopping problems in continuous hidden Markov models. " *Stochastics: An International Journal of Probability and Stochastic Processes*, vol. **94**(4), pp. 335-364.

[2] Leland H. E. (1994), "Corporate debt value, bond covenants, and optimal capital structure. " *Journal of Finance*, vol. **49**(4), pp. 1213-1252.

[3] Peskir G. (2007), "A change-of-variable formula with local time on surfaces. " *Séminaire de Probabilité XL*, *Lecture Notes in Mathematics*, vol. 1899, Berlin: Springer, pp. 69-96.

[4] Peskir G., Shiryaev A. N. (2006), "*Optimal Stopping and Free-Boundary Problems*, " Birkhäuser: Basel.