Local Risk Minimization for American Options under Stochastic Early Exercise and Transaction Costs

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Abstract

This paper studies local risk-minimization (LRM) hedging of American put options in a setting where early exercise is only partially predictable and trading is subject to transaction costs. To bridge the gap between the classical optimal-stopping paradigm and heterogeneous real-world exercise behavior, we model the holder's decision as a state-dependent random stopping rule driven by moneyness through an Erlang distribution. Within this random-exercise framework, we derive discrete-time LRM recursions for the option value and the optimal hedge ratio under both proportional and fixed transaction costs. The resulting hedge updates are implicit; we establish conditions guaranteeing uniqueness and compute the strategy efficiently using a willow tree lattice combined with bisection at each node.

Our numerical results highlight several robust implications. First, transaction costs materially affect both valuation and hedging: LRM prices increase relative to the frictionless benchmark, and fixed costs generate larger distortions than proportional costs, particularly when hedging is frequent. Second, the interaction between transaction costs and hedging frequency is central. With fixed costs, less frequent rebalancing can reduce the total trading burden and alter the optimal hedge aggressiveness, while proportional costs yield a different cost-risk balance. Third, model choice remains economically important under realistic exercise behavior. Across GBM, Merton jump-diffusion, and GARCH specifications, we observe meaningful differences in hedge profiles and in the distributional properties of hedging errors, especially in the tails. Simulation-based performance assessment confirms that early-exercise uncertainty, cost structure, and jump/volatility dynamics jointly shape hedging risk, so the classical rebalance more often vs. pay more costs trade-off cannot be evaluated in isolation.

Keywords: Local risk minimization hedge, American option, random exercise, transaction cost, willow tree method.

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