A characterization of ruin-inducing probability measures in a renewal risk model

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Abstract

In classical risk theory, explicit formulas for ruin probabilities are rarely available. As a consequence, the evaluation of the infinite-time ruin probability $\psi(u)$, $u \in \mathbb{R}_+$, typically relies on simulation techniques. While the crude Monte Carlo method can be applied to finite-time ruin probabilities, it cannot in general be used directly to estimate $\psi(u)$, since the event $\{\tau_u < \infty\}$, where τ_u denotes the ruin time, cannot be simulated within finite time.

A classical approach to overcome this difficulty is based on the Esscher change of measures, enabling the construction of a probability measure Q under which ruin occurs almost surely (a.s.) while preserving the probabilistic structure of the model. This approach has been widely used in the literature to tackle various ruin-related problems as it resolves the infinite time horizon problem and allows $\psi(u)$ to be expressed as an expectation under Q, making simulation feasible. However, the construction of Q fundamentally relies on the existence of the moment generating function (mgf) of the claim size distribution and therefore excludes heavy-tailed distributions. In such settings, alternative simulation approaches are typically based on the Pollaczek-Khinchin formula; however, these are primarily applicable to the Cramér-Lundberg model, where the ladder height distribution admits an explicit form.

Given a compound renewal process S defined on a probability space (Ω, Σ, P), we establish, under mild assumptions, a complete characterization of the class of probability measures Q on Σ that are progressively equivalent to P, preserve the probabilistic structure of S, and under which ruin occurs a.s.. The characterization is expressed in terms of suitable pairs of tilting functions (γ, δ) acting on the claim size and interarrival time distributions. This framework allows the construction of general exponential martingales without assuming the existence of the mgf of the claim size distribution, leading to a general representation of $\psi(u)$ suitable for simulation. The proposed approach includes the classical Esscher transform as a special case and extends naturally to heavy-tailed claim size distributions.

Keywords: Compound renewal process; progressively equivalent measures; change of measures; exponential martingales; ruin probability

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