Optimal Design of Model-Contingent Insurance Contracts

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Abstract

We revisit the problem of optimal insurance design when the loss distribution is ambiguous but identifiable. As insurance companies typically charge a premium that is larger than the actuarial value of the contract, a complete risk transfer is suboptimal. Arrow [1] famously showed that the optimal insurance contract contains a straight deductible when the loss distribution is known. In general, this result does not carry over to the case of ambiguous loss distributions, and only holds true when the ambiguity takes a very specific form ([5], [6]). We show that Arrow's result extends to ambiguous settings if ambiguity is identifiable in the sense of Denti and Pomatto [4] and the policyholder evaluates ambiguity using the smooth model ([3]).
We consider a policyholder, endowed with initial wealth w, who faces the risk of a loss X, a positive random variable defined on a state space Ω. The distribution of X is ambiguous, and the policyholder considers a set

$$\mathcal{P} = \{P_{\theta_i} \mid \theta_i \in \Theta\}$$

of loss distributions (or models), parametrized by $\theta_i \in \Theta$, where Θ is a countably finite parameter set. The probability weights of models are given by $q = (q_{\theta_1}, ..., q_{\theta_n})$.
We interpret uncertainty about the loss distribution as model uncertainty, where each possible loss distribution is shaped by objective parameters and mechanisms that can be estimated ex-post. This requires identifiability of loss distributions, i.e., there exists a measurable function $k : \Omega \to \mathcal{P}$ such that $k(\omega) = P_{\theta_i}$ P_{θ_i}-a.s. ([2], [4]).
We show that identifiability enables the insurer to offer contracts that are model-contingent. An insurance contract is defined as a pair $((I_{\theta_i})_{\theta_i \in \Theta}, \pi)$, specifying model-contingent indemnity schedules I_{θ_i} and the premium π. That is, $I_{\theta_i}(x)$ specifies the amount paid by the insurer if the realized loss is x and the identified model is P_{θ_i}.
We adopt the identifiable smooth model of decision making under ambiguity ([3], [4]). The policyholder maximizes wealth given the premium calculation of the insurer

$$\max_{(I_{\theta_i})_{\theta_i \in \Theta}} \sum_{i=1}^{n} q_{\theta_i} \phi \left(\int u(w - X + I_{\theta_i}(X) - \pi(I)) dP_{\theta_i} \right) \tag{1}$$

$$\text{with } \pi(I) = (1+\tau) \left(\sum_{i=1}^{n} q_{\theta_i} \int I_{\theta_i}(X) dP_{\theta_i} \right), \tag{2}$$

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where u and ϕ govern the policyholder's risk and ambiguity attitude, respectively. The insurer is risk and ambiguity neutral and applies the expected value principle with loading factor (or transaction cost) $\tau \geq 0$.

By applying methods from calculus of variations, we show that a unique solution to Problem (1) exists. We then show that this solution takes the form of straight deductibles, extending Arrow's cornerstone result to the case of ambiguous loss distributions. In particular, we show that there exists $D = (D_{\theta_i})_{\theta_i \in \Theta}$ such that optimal model-contingent indemnity schedules are given by

$$I^*_{\theta_i}(x) = \max(x - D_{\theta_i}, 0).$$

Building on this result, we show that full insurance in each model is bought if and only if the premium is actuarially fair. Given an actuarially unfair premium, we further discuss the ordering of optimal deductibles D in dependence of the stochastic order of loss distributions. We show, given that models are ranked according to first order stochastic dominance, the policyholder opts for higher coverage in worse models, while this is not necessarily the case when models can only be ranked by second order stochastic dominance.

Lastly, we provide comparative statics of the policyholder's ambiguity aversion. An ambiguity-neutral policyholder opts for a model-independent deductible, while ambiguity aversion causes optimality of model-contingent deductibles.

Keywords: Insurance design, model uncertainty, smooth ambiguity aversion, identifiability, straight deductibles

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