A Geometric Process Approach to Modelling Mortality Jumps in the Lee–Carter Model Framework

Selin ÖZEN*[1]

[1]Department of Actuarial Sciences, Faculty of Applied Sciences, Ankara University, Ankara, TURKEY

Abstract

Recent catastrophic events such as the COVID-19 pandemic have exposed important limitations of traditional mortality models in capturing the evolving frequency and timing of extreme mortality events. While existing extensions of the Lee–Carter model (1992) framework incorporate transitory or permanent jump components, they typically assume constant or memoryless jump-arrival mechanisms. Such assumptions restrict the ability of these models to reflect structural changes in the recurrence of catastrophic mortality shocks [4].

This study proposes a novel mortality modelling framework by integrating a *Geometric Process* (GP) into the Lee–Carter model to govern the inter-arrival times of mortality jumps. The GP generalises the traditional renewal process by introducing a trend-sensitive ratio parameter a, which controls the monotonic expansion or contraction of inter-arrival times [2]. Unlike the classical renewal process, which assumes identically distributed inter-arrival times, the geometric process allows waiting times between jumps to evolve monotonically over time through a ratio parameter a. This feature enables the model to capture trend-dependent jump frequencies, reflecting changing epidemiological, demographic, and environmental conditions.

The proposed model, referred to as the LCG model, is evaluated against two benchmark specifications: the transitory jump model of Chen and Cox (2009) and the renewal-process-based LCR model of Özen and Şahin (2020) ([1]; [4]). Mortality data from Denmark, Spain, Sweden, and Switzerland are analysed using maximum likelihood estimation. Several candidate distributions for inter-arrival times are examined, and the lognormal distribution is found to provide the best empirical fit within the geometric process framework across all countries.

In-sample model comparisons based on the Bayesian Information Criterion (BIC), Mean Absolute Percentage Error (MAPE), and Root Mean Square Error (RMSE) demonstrate that the LCG model consistently outperforms the benchmark models. The improvement arises from the geometric process ability to capture trend-sensitive jump frequencies and semi-persistent mortality dynamics.

---

*E-mail address: ozens@ankara.edu.tr

Out-of-sample forecasting performance is assessed using post-2019 mortality data, covering the COVID-19 period. The results show that the LCG model yields the lowest forecast errors across all countries, confirming its superior ability to capture post-pandemic mortality dynamics.

Finally, the model is embedded within a Solvency II framework to compute Solvency Capital Requirements (SCR) using a Value-at-Risk approach at the 99.5% confidence level. The LCG model produces more balanced and lower SCR values compared to alternative models, indicating improved capital efficiency without understating risk. Overall, the proposed framework offers a flexible and realistic approach to modelling mortality jumps, with important implications for actuarial forecasting, risk management, and solvency assessment.

Keywords: transitory mortality jumps, geometric process, extreme mortality risk, mortality modelling.

References

[1] H. Chen and S.H. Cox (2009), “Modeling Mortality with Jumps: Applications to Mortality Securitization ” *The Journal of Risk and Insurance*, vol. **76** (3), pp. 727-751.

[2] Y. Lam (1988), “Geometric Processes and Replacement Problem ” *Acta Mathematicae Applicatae Sinica*, vol. **4**, 366-377, pp. 1988.

[3] R.D. Lee and L.R. Carter (1992), “Modeling and Forecasting U.S. Mortality ”*Journal of the American Statistical Association*, vol. **87**, pp. 659–671.

[4] S. Özen and Ş. Şahin (2020), “Transitory Mortality Jump Modeling with Renewal Process and Its Impact on Pricing of Catastrophic Mortality Bonds ” *Journal of Computational and Applied Mathematics*, vol. **376**, pp. 1-15.