Asymptotics of Ruin Probabilities in a Subordinated Cramér-Lundberg Model

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Abstract

Insurers today face a wide range of risks that need to be modeled appropriately. On the one hand, there are individual risks and associated individual losses, such as vehicle damage covered by comprehensive motor insurance in the case of an accident. On the other hand, there are natural hazards such as hail, storms, or earthquakes. In these cases, the insurer records cumulative losses arising from many individual claims that occur simultaneously (e.g., hail damaging numerous vehicles at once). Due to climate change, natural catastrophes and their financial consequences have gained increasing relevance, making the modeling of cumulative risks more important than ever.

Cumulative losses caused by natural disasters have a substantial impact on the insurer's probability of ruin, as they occur at a single point in time rather than being spread over the duration of the insurance contract, as is typical for independent individual losses. In this paper, we construct a model that captures the effect of cumulative losses on the probability of ruin. We consider a compound Poisson process describing the insurer's loss side and introduce a Lévy subordinator that acts as a stochastic time change of this process. This time change allows the base process to 'jump' over certain periods, so that what are individual losses in ordinary time may cluster into a large aggregate loss in the time-changed process. The subordinated loss process therefore exhibits random clustering of individual claims. The model extends [3], though we focus on a different application.

To ensure comparability between the subordinated process and the original compound Poisson process, we require the subordinator to run on average at the same speed as calendar time. Consequently, the expected value of the compound Poisson process remains unchanged under subordination. We demonstrate that subordination invariably enlarges the jump size distribution of the compound Poisson process in the sense of stochastic dominance (see also [1]) and that, through the clustering of individual jumps, subordination can transform light-tailed jump distributions into heavy-tailed ones.

We derive conditions under which the modified jump distribution has light tails, allowing the application of classical Cramér-Lundberg arguments. In the light-tailed case we prove, via the Adjustment Coefficient, that subordination asymptotically increases the ruin probability.

We then turn to the heavy-tailed case and identify conditions under which the modified jump

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distribution becomes heavy-tailed. Our analysis focuses on subexponential distributions, in particular those with regularly varying tails, and using [2] we derive conditions for the subordinated process to inherit these tail properties.

Keywords: Cramér-Lundberg Model, Ruin-Theory, Subordination, Subexponential Distribution, Regular Variation.

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