

Optimal Protective Investment in the Classical Risk Model

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Abstract

Insurance companies employ various strategies to reduce expected claims expenditure. While the transfer of risk to reinsurers to reduce the claim size remains the most prominent instrument, insurers increasingly invest in preventive measures, such as financing medical check-ups in health insurance or occupational safety training in accident and disability insurance, in order to reduce the expected number and severity of claims.

In this paper, we propose a risk model that incorporates dynamic protective investment. The insurer's surplus process is modeled by a classical Cramér-Lundberg process, where the insurer has the option to allocate part of the premium income to protective measures. The aim of protective investment is to reduce the expected number of claims as well as the claim size.

We formulate an infinite time-horizon stochastic control problem in which the insurer aims to minimize the probability of ruin by dynamically choosing the protective investment. The claim arrival intensity as well as the claim severity is assumed to be a decreasing and convex function of this investment.

We first provide a rigorous mathematical formulation of the control problem and admissible strategies. We then derive the associated Hamilton-Jacobi-Bellman (HJB) equation and prove the existence and uniqueness of a solution satisfying the appropriate boundary conditions. Moreover, we establish that the value function of the control problem coincides with this solution to the HJB equation.

Our analysis reveals that when there is only an effect on the intensity of claim arrivals, the optimal protective investment strategy is constant in the state. In contrast, when protective investment affects both the intensity and severity, the optimal prevention level becomes state-dependent. We compute the optimal protective investment strategy numerically and observe very different structures of the optimal strategy depending on the model choice. Moreover, we numerically explore the asymptotic behavior of the optimal strategy for both light- and heavy-tailed claim size distributions and discuss its relation to classical asymptotic results for

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the Cramér-Lundberg model.

The proposed model can be interpreted as a dynamic extension of the model from [1], augmented by an additional effect on the claim severity. In a special case, by choosing appropriate model parameters, our model is equivalent to an optimal proportional reinsurance model, as analyzed in [2].

Keywords: Cramér-Lundberg; HJB-Equation; Optimal Control; Protection; Ruin Theory.

References

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