INFINITE-TIME RUIN PROBABILITY OF A MULTIVARIATE RENEWAL RISK MODEL WITH BROWNIAN PERTURBATIONS

DIMITRIOS G. KONSTANTINIDES

Keywords: Ruin probability; Integrated tail; Multivariate subexponentiality; Multivariate regular variation; Brownian motion;
Mathematics Subject Classification: Primary 62P05; Secondary 91G05; 91B05

The problem of insurer's ruin in presence of heavy tailed distributions for the claim sizes is well-known already from the last decade of previous century, see in [2], [1], [7] for some monographs on this topic. Given that the modern insurance companies are forced to keep more than one risky portfolios, to face the ruin event, as also the dependence structures among the portfolios, the attention of an increasing number of researchers is focused on multivariate risk models with heavy-tailed distribution of claim sizes, see for example [5], [4], [6], [8], [11], [13], [10], [12], [9] among others.

In this paper, we consider an insurer who operates d-lines of business, with $d \in \mathbb{N}$, and these lines share a common counting process. Let us assume that the claim vectors $\{\mathbf{X}^{(i)}\,,\ i \in \mathbb{N}\}$ follow distributions, whose support is included on the nonnegative half-axis, and each claim vector $\mathbf{X}^{(i)} = \left(X_1^{(i)}\,, \ldots,\, X_d^{(i)}\right)^{\top}$ may has zero components, but not all of them equal to zero. Let us assume that the claim vectors $\{\mathbf{X}^{(i)}\,,\ i \in \mathbb{N}\}$ arrive at the moments $\{\tau_i\,,\ i \in \mathbb{N}\}$, with $\tau_0 = 0$, that constitute a counting process $\{N(t)\,,\ t \geq 0\}$, defined as follows

$$N(t) = \sup\{n \in \mathbb{N}\ :\ \tau_n \leq t\}\,,$$

for any $t \geq 0$.

Hereafter, we suppose that $\{N(t)\,,\ t \geq 0\}$ is a renewal process, namely the sequence of the inter-arrival times $\{\theta_i = \tau_i - \tau_{i-1}\,,\ i \in \mathbb{N}\}$ is sequence of independent and identically distributed (i.i.d.) nonnegative random variables.

Further, we suppose that the insurer charges premiums from the d lines of business, whose rate of payments is described by the deterministic vector $\mathbf{p} = (p_1\,, \ldots,\, p_d)^{\top}$, with $p_i \in (0,\, \infty)$, for any $i = 1,\, \ldots,\, d$, while he keeps initial capital $x > 0$, that is allocated on the d lines of business according to $\mathbf{b} = (b_1\,, \ldots,\, b_d)^{\top}$, with $b_i > 0$, for any $i = 1,\, \ldots,\, d$, and $b_1 + \cdots + b_d = 1$.

Finally we add one more source of randomness, which stems either from premiums or from claims, described by a multivariate Brownian motion $\{\mathbf{B}(t) = (B_1(t)\,, \ldots,\, B_d(t)\,)^{\top}\ ,\ t \geq 0\}$ that has arbitrarily correlated components. Hence, if $\vec{\delta} \geq \mathbf{0}$ is a fixed vector, named diffusion

---

Date: March 5, 2026.

coefficient, the insurer's surplus process at moment $t > 0$ can be described by the equation

$$\begin{aligned}
&\mathbf{U}(t) \\
&:= \begin{pmatrix} U_1(t) \\ \vdots \\ U_d(t) \end{pmatrix} = x \begin{pmatrix} b_1 \\ \vdots \\ b_d \end{pmatrix} + t \begin{pmatrix} p_1 \\ \vdots \\ p_d \end{pmatrix} - \begin{pmatrix} \sum_{i=1}^{N(t)} X_1^{(i)} \\ \vdots \\ \sum_{i=1}^{N(t)} X_d^{(i)} \end{pmatrix} + \begin{pmatrix} \delta_1 \\ \vdots \\ \delta_d \end{pmatrix} \odot \begin{pmatrix} B_1(t) \\ \vdots \\ B_d(t) \end{pmatrix} \\
&= x\,\mathbf{b} + t\,\mathbf{p} - \sum_{i=1}^{N(t)} \mathbf{X}^{(i)} + \vec{\delta} \odot \mathbf{B}(t)\,, \qquad (0.1)
\end{aligned}$$

where by $\odot$ is denoted the Hadamard product. In risk model (0.1) we make the following assumption.

Assumption 0.1. *The sequence $\{\mathbf{X}^{(i)}\,,\ i \in \mathbb{N}\}$ contains i.i.d. random vectors with common distribution F, and they are independent of the sequence of arrival moments $\{\tau_i\,,\ i \in \mathbb{N}\}$. Further we assume that $\{\mathbf{B}(t)\,,\ t \geq 0\}$ is independent from all other sources of randomness and has nonnegative expectation, in the sense $\mathbf{E}[B_j(t)] \geq 0$, for any $j = 1, \ldots, d$ and any $t \geq 0$.*

References

[1] Asmussen, S., Albrecher, H. (2010) *Ruin Probabilities.* World Scientific, Singapore, 2nd ed.

[2] Embrechts, P., Klüpellberg, C. and Mikosch, T. (1997) *Modelling Extremal Events for Insurance and Finance.* Springer, New York.

[3] Embrechts, P., Omey, E. (1984) A property of Longtailed distributions. *J. Appl. Probab.*, **21**, 80–87.

[4] Gao, Q., Yang, X. (2014) Asymptotic ruin probabilities in a generalized bidimensional risk model perturbed by diffusion with constant force of interest. *J. Math. Anal. Appl.*, **419**, no. 2, 1193–1213.

[5] Hult, H., Lindskog, F. (2006) Heavy-tailed insurance portfolios: buffer capital and ruin probabilities. *Technical report.*

[6] Jiang, T., Wang, Y., Chen, Y., Xu, H. (2015) Uniform asymptotic estimate for finite-time ruin probabilities of a time-dependent bidimensional renewal model. *Insur. Math. Econom.*, **64**, 45–53.

[7] Konstantinides, D.G. (2018) *Risk Theory. A Heavy Tail Approach.* World Scientific, New Jersey.

[8] Konstantinides, D.G., Li, J. (2016) Asymptotic ruin probabilities for a multidimensional renewal risk model with multivariate regularly varying claims. *Insur. Math. Econom.*, **69**, 38–44.

[9] Konstantinides, D.G., Liu, J., Passalidis, C.D. (2026) Uniform asymptotics for a multi-dimensional renewal risk model with multivariate subexponential claims. *Scand. Actuar. J.*, 1–21. DOI:10.1080/03461238.2025.2584008.

[10] Konstantinides, D.G., Passalidis, C.D. (2025) Uniform asymptotic estimates for ruin probabilities of a multidimensional risk model with cádlág returns and multivariate heavy-tailed claims. *Insur. Math. Econom.*, **125**, 103148.

[11] Li, J. (2016) Uniform asymptotics for a multi-dimensional time-dependent risk model with multivariate regularly varying claims and stochastic return. *Insur. Math. Econom.*, **71**, 195–204.

[12] Passalidis, C.D. (2025) Multivariate strong subexponential distributions: Properties and Applications. *Preprint, arXiv:2503.22267.*

[13] Samorodnitsky, G., Sun, J. (2016) Multivariate subexponential distributions and their applications. *Extremes*, **19**, no. 2, 171–196.

[14] Yuan, M., Lu, D. (2023) Asymptotics for a time-dependent by-claim model with dependent subexponential claims. *Insur. Math. Econom.*, **112**, 120–141.

University of the Aegean, Karlovassi, GR-83 200 Samos, Greece
Email address: konstant@aegean.gr.