Stochastic Control of Dividends with a Drawdown Penalty

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Abstract

We consider a diffusion risk model of the form

$$X^U(t) = x + \mu t + \sigma W(t) - \int_0^t U(s)\,\mathrm{d}s\;,$$

where dividends are paid at rate $U(t) \in [0, u_0]$. In order to measure the stability of the surplus process, we are interested in the so-called *drawdown*, that is the distance between the current capital and the last historical maximum:

$$\Delta_z^U(t) = \max\{z, \sup_{s\in[0,t]} X^U(s)\} - X^U(t)\;.$$

Our aim is now to maximise dividend payments while minimising the time in which the drawdown size exceeds a critical level $d > 0$. This idea is represented by the *value function*

$$v(z) = \sup_U \mathbb{E}\Big[\beta \int_0^\infty \mathrm{e}^{-rt}\,U(t)\,\mathrm{d}t - \int_0^\infty \mathrm{e}^{-rt}\,\mathbf{1}_{\{\Delta_z^U(t)>d\}}\,\mathrm{d}t\Big]\;,\qquad z \ge 0\;.$$

The problem is connected to the Hamilton–Jacobi–Bellman equation

$$-rh(z) - \mu h'(z) + \frac{\sigma^2}{2}h''(z) + \sup_{u\in[0,u_0]}\{(\beta + h'(z))u\} = \mathbf{1}_{\{z>d\}}\;. \tag{1}$$

We show that the value function is the unique solution to (1) and derive a semi-explicit expression by solving a system of differential equations. It turns out that an optimal dividend strategy is of feedback form, where the optimiser is given by

$$u^*(z) = u_0\mathbf{1}_{\{v'(z)\ge-\beta\}}(z) = u_0\mathbf{1}_{[0,z_f]\cup[z_g,\infty)}(z)$$

for some $0 \le z_f \le d \le z_g < \infty$. We provide some numerical examples to illustrate our results.

Keywords: DRAWDOWN; DIFFUSION APPROXIMATION; OPTIMAL DIVIDENDS; HAMILTON–JACOBI–BELLMAN EQUATION

References

[1] Dudziak, K. and Schmidli, H. (2025), "Stochastic Control of Dividends with a Drawdown Penalty." *Preprint.* arXiv:2510.25494 [math.OC].

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