# Pricing of geometric Asian options in the Volterra-Heston model

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## Abstract

Asian options are widely used in insurance-linked products and structured finance to mitigate short-term market fluctuations by averaging the underlying asset price over time. While closed-form pricing results for geometric Asian options are available in the Black–Scholes framework and in the classical Heston model, explicit formulas are largely missing in settings where volatility exhibits memory effects or rough behavior, as documented empirically in financial markets [3].

In this contribution, we address this gap by studying the pricing of fixed- and floating-strike geometric Asian options in the class of Volterra-Heston models. These models describe stochastic volatility via a square-root Volterra equation and naturally incorporate non-Markovian dynamics, including rough volatility as a special case. Our analysis is firmly rooted in the theory of *affine Volterra processes*, cf. [1], which provides a rigorous extension of classical affine diffusions to non-Markovian settings while preserving analytical tractability.

**Main contributions of this work are:**

- We formulate a model-independent pricing framework for geometric Asian options based on the conditional joint Fourier transform of the log-asset price at maturity and the logarithm of the geometric average, extending the Markovian approach of [4] to non-Markovian volatility models.
- By embedding the Volterra-Heston model into the class of affine Volterra processes as e.g. given in [1], we derive an explicit exponential representation of this joint Fourier transform in terms of the forward variance process and the solution of a Riccati-Volterra equation.
- We obtain semi-closed pricing formulas for fixed- and floating-strike geometric Asian call and put options, generalizing the classical Heston results and recovering them as a special case.

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- A numerical study in the rough Heston setting demonstrates that volatility roughness and memory effects have a pronounced impact on Asian option prices, especially for longer maturities, which is particularly relevant for long-dated insurance and financial contracts.

From an actuarial perspective, our results provide analytically tractable pricing tools for path-dependent derivatives in volatility models with memory, highlighting the central role of affine Volterra processes in modern option pricing.

This talk is based on [2].

**Keywords:** Asian options ; Volterra-Heston model; Rough volatility; Affine Volterra processes; Option pricing

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# References

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