Title: Reconciling P- and Q-Calibration: The Discrete-Time 4-Factor Path-Dependent Volatility Model

Abstract:

Model calibration under P and under Q are often regarded as two separate branches of finance. P refers to a backward-looking real-world probability measure under which the observed historical price path of a financial asset is viewed as a realization of a stochastic process, while Q denotes a forward-looking risk-neutral measure inferred from the prices of options written on this underlying asset. Accordingly, model estimation based on past prices of the underlying asset is referred to as estimation "under P", while estimation from option prices is known as calibration "under Q". One may question whether such a strict separation is justified or whether it rather reflects the lack of models able to capture the joint dynamics of prices and implied volatilities. Path-dependent volatility models are uniquely positioned to reconcile P- and Q-calibration, since they precisely relate past asset returns to volatility, thus to option prices. In this talk, we introduce the discrete-time 4- (or 3-)factor path-dependent volatility model and we show that combining the path-dependency of volatility that we uncovered in the article *Volatility Is (Mostly) Path-Dependent* (Guyon and Lekeufack, 2023) with fat-tailed random innovations allows us to reconcile model calibration under P and under Q, which further supports the hypothesis of high endogeneity of volatility. We also propose a new estimation approach that combines P- and Q-information to enhance calibration robustness, and we benchmark its effectiveness against classical methods. This is joint work with Léo Parent.