Closure properties and heavy tails: random vectors in the presence of dependence

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Abstract

This paper is organized in three parts closely related to closure properties of heavy-tailed distributions and heavy-tailed random vectors. In the first part we consider two random variables X and Y with distributions F and G respectively. The two supports of these functions coincide with $[0, \infty)$, although in some cases the support of F can be extended to the whole real axis. We assume that these random variables satisfy one type of a weak dependence structure. Under some mild conditions, we examine whether their product convolution distribution H belongs in the same distribution class of the distribution F. Namely we establish the closure property with respect to the product convolution, under this specific weak dependence structure, in the classes $\mathcal{ERV}, \mathcal{C}, \mathcal{D}, \mathcal{M}, \mathcal{M}^*, \mathcal{OS}, \mathcal{OL}, \mathcal{P}_{\mathcal{D}}$ and $\mathcal{K}$. Further in the second part we introduce a new distribution class, which satisfies some closure properties such as product convolution convolution and mixture. Although the multivariate regular variation is well-established distribution, it does not happen in other heavy tailed random vectors. Therefore in the third part we introduced the class of dominatedly varying vectors and positively decreasing random vectors and we study the closure property of the scalar product under independent and dependent cases. Furthermore we study the closure property of the first class under convolution and mixture, and we study the distribution of stopped sums where the summands are random vectors which belongs to this class. Some of these results holds and for positively decreasing random vectors.

Keywords: product convolution; long positive decrease; dominatedly varing random vectors; positively decreasing random vectors

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