Metric embeddings of tail correlation matrices

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Abstract

The assessment of risks associated with multivariate random vectors $(X_1, \ldots, X_n)$ relies heavily on understanding their extremal dependence, crucial in evaluating risk measures for financial or insurance portfolios. A widely-used metric for assessing tail risk is the tail dependence matrix of tail correlation coefficients, given by (if the limits exist)

$$\mathbf{\Lambda} = (\lambda_{i,j})_{i,j\in\{1,\ldots,n\}} := \left(\lim_{u\nearrow 1} P(F_i(X_i) > u | F_j(X_j) > u)\right)_{i,j\in\{1,\ldots,n\}},$$

where F_i denotes the marginal distribution function of $X_i, i = 1, \ldots, n$. Among the exploration of structural properties of $\mathbf{\Lambda}$ the so-called realization problem of deciding whether a given matrix is the tail correlation matrix of *some* $(X_1, \ldots, X_n)$ has recently received some attention, see [2, 3, 5].

The entries of the matrix $\mathbf{\Lambda}$ are closely related to a useful distance measure on the space of Fréchet(1)-random variables, named spectral distance and first introduced in Davis & Resnick (1993). We analyze the properties of the related semimetric d on a set of n elements $\{1, \ldots, n\}$ defined by $d(i, j) := 1 - \lambda_{i,j}, i, j \in \{1, \ldots, n\}$, and show that it is both L_1- and ℓ_1-embeddable. Notably, these embeddings bear a direct relationship with the realization of specific tail dependence structures via max-stable random vectors. Particularly, an ℓ_1-embedding of d, employing line metrics, provides a representation through a max-stable mixture of so-called Tawn-Molchanov models.

Leveraging this framework, we revisit the realization problem, affirming a conjecture by [5] regarding its NP-completeness.

This talk is based on the paper [4].

Keywords: Bernoulli compatibility, Line metrics, Max-stable random vectors, Metric embedding, Multivariate regular variation, NP completeness, Tail-dependence (TD) matrix, Tail correlation coefficients, Tawn-Molchanov models, Realization problem for TD matrices

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