On the Expansion of Risk Pooling

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Extended abstract

Risk pooling has been an increasingly critical tool for managing catastrophe risks between corporations and among nation states with examples including multinational pooling and catastrophe risk pooling. At the same time, risk-sharing schemes among individuals are becoming famous (e.g. P2P settings), as part of the so-called decentralized insurance (see Abdikerimova and Feng 2022 and Denuit and Robert 2021). While there has been a rich literature on such practices, little is known from a theoretical viewpoint regarding the operational strategies of risk pools and in particular on issues about the effect of a risk-sharing group's expansion on each existing members and whether it could cause the exit of some of them.

The paper is the first to study this kind of issues by establishing different notions of consensus for the expansion of a risk pooling. We define the notion of *strong consensus (SC)* of a sharing pool according to which both the existing members and the candidate improve their risk measures due to pool's expansion. On the other hand, the *weak consensus (WC)* refers to the existing members and holds when none of them is willing to leave the pool due to its expansion. The SC is linked to ballot-box-voting settings where each member votes to accept a candidate (where veto could also be applied), while the WC is connected to voting-by-feet situations (where a candidate is always allowed to join but any existing member can leave).

At any of the possible pools the risk-sharing consists of two parts: the sharing of individual risks, where we apply Pareto optimal sharing rules; and the allocation of internal cash transfers which are placed in order to make the sharing fair. For the latter, we employ two distinct pricing rules: the *actuarial fairness principle (AFP)*, which prices risks as their expected values and the *equilibrium pricing (EQP)*, which is the pricing mechanism that clears out the market (considering risks as assets as in Anthropelos and Žitković 2010a and Anthropelos and Žitković 2010b). As for the agents' risk preferences, we impose the entropic risk measure, which not only implies linear optimal sharing rules (see Barrieu and El Karoui 2005), but also helps to derive analytic formulas with clear economic messages.

The paper draws a number of interesting insights: Risk pools tend to be conservative under the AFP and risk-seeking under the EQP. In the former, consensus conditions are solely determined by the so-called deviation-tolerance ratio, which can be viewed as an uncertainty level relative to the participant's tolerance. Comparison of deviation-tolerance ratios in the expanded pool with that in the status-quo or in a stand-alone state yields the parameterized conditions' characterization. In fact, we show that the existing group prefers a candidate of low deviation-tolerance ratio.

The situation is quite different under EQP. Since each agent's demand at equilibrium is optimal, weak consensus always holds and each candidate is willing to enter the group. This is an important feature when the expansion of a group is the main goal, since none wants to leave and all want to enter. The SC considers the improvement or not of the existing members' risk measure due to expansion. Under EQP, the existing

members prefer candidates with higher risks and the reason for this is that equilibrium pricing penalizes higher risk with higher premium that is allocated among the existing members. In other words, EQP contains a "profit loading" component in the sense that existing members, besides sharing risk, play the role of an insurer for the highly risky candidates. This implies that EQP creates a ground for pool that could handle large risks.

We also examine the employment of reinsurance on sharing pool, in a classic setting where the reinsurer covers all the aggregate risk above a pre-specified level. Although, reinsurance improves the risk measure of the group as a whole, it makes the risk sharing more appealing for candidates under AFP than EQP. In the context of a strong consensus, reinsurance does not change the acceptance decision of existing members under the AFP. In contrast, under EQP, a candidate is less likely to be accepted with a reinsurance when he brings high risk (since the insurance of his risks is going to be split to both reinsurer and the existing members).

Keywords: Risk-sharing, expansion of sharing, decentralized insurance, sharing of catastrophic risks, P2P.

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