

# Closure properties in positively decreasing and related distributions under dependence

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## Abstract

We consider closure properties in the class of positively decreasing distributions. Our results stem from different types of dependence, but each type belongs in the family of asymptotically independent dependence structure. Namely we examine the closure property with respect to minimum, maximum, convolution product and convolution. Furthermore, we take into account some closure properties of the class of generalized subexponential positively decreasing distributions, as also we introduce and study the class of the generalized long-tailed positively decreasing distributions. Also we revisited the (independent) convolution closure problem of subexponentiality, in the case of subexponential positively decreasing class. In some classes we discuss the closedness of randomly stopped sums. In the last section we study the closure property with respect to minimum for two classes of random vectors.

*Keywords:* asymptotic analysis, heavy-tailed distributions, heavy-tailed random vectors, asymptotic independence, subexponentiality.

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