Optimal consumption, investment, and life insurance including state dependence by risk-adjusted utility

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Abstract

We investigate optimal consumption, investment, and life insurance problems, as formulated by [7], by incorporating state dependence, which typically results in an expansion of state variables, consequently leading to a more intricate problem. We work with an approach to state dependence that streamlines the problem rather than complicates the results and solutions. [8] initially proposed that within the classical [6, 5] framework, an agent's preferences factor in the market's pre-established risk attitude, assuming that the agent's decisions do not influence the market's risk perception. Further, building on the findings and explorations of [3], [2], and most recently, [4], we include the risk associated with life insurance so the agent can factor in both the attitude to risk that the financial and the insurance market have already made up its mind.

The idea is to quantify the financial worth of payments and compare these. Instead of evaluating the utility and, consequently, the moral value of the money, we take cues from derivative pricing techniques and compare payments by price or financial value. In a complete market that takes a unique position on the price of the risk associated with the financial market's risk and the price of the risk associated with the actuarial market, we consider the risk behind financial securities such as longevity bonds. Working with a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0\leq t\leq T}, \mathbb{P})$ that is home to a market containing a constant risk-free interest rate r in which a financial asset and an actuarial asset (the longevity bond) are traded. In this setting, we know from [1] and arbitrage pricing theory that the price or financial value at time 0 of an $\mathcal{F}$-measurable payment of $u'(t)$ at time t is given by $\mathbb{E}[\Lambda(t)u'(t)]$, where Λ is a deflator that adjusts for the risk associated with insurance by incorporating mortality credits, representing the difference between the pricing and objective mortality rates, represented by the function g in the deflator. By including life insurance in the deflator, the financial value of payments is risk-adjusted not only for financial risk but also for the risk of mortality.

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In the risk-adjusted framework, we present and interpret solutions to the optimal consumption, investment, and life insurance problem for general utility functions and CRRA (constant relative risk aversion) power utility functions. Further, we explore specific corner cases to illustrate results and interpretations of the optimal controls and their dynamics. These solutions are intuitively reasonable strategies for consumption, investment, and life insurance under power utility and allow us to understand the framework and the solutions.

Keywords: Optimal consumption investment and life insurance, state-dependent utility, Hamilton-Jacobi-Bellman equation.

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