Investigating trade-offs in the design of smooth pension products

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Abstract

We investigate different ways to design smooth pension products based on solutions from optimal consumption and investment problems. Smoothness of a consumption process can be studied from both a pathwise (measured in terms of quadratic variation) and a pointwise (measured in terms of variance) point of view and we conclude that introducing one type of smoothing does not necessarily improve the other type of smoothing. Thus care has to be taken when designing smooth pension products.

Focusing on pathwise smoothness without disregarding pointwise smoothness, we provide both a qualitative as well as a quantitative discussion of the trade-offs involved. In the qualitative discussion we find that to increase smoothness it is necessary either reduce the starting value, the drift of consumption or the level of terminal wealth.

For the quantitative discussion we set up an optimal consumption and investment problem, where the first control is the proportion of wealth invested into the risky asset, but the second control is not the consumption process itself. Instead we use the drift and volatility of consumption as controls. The objective is to minimise the quadratic distance to a target drift and volatility, while introducing a penalty on the volatility. We find explicit solutions to this problem using classic dynamic programming methods and use them to study the three trade-offs theoretically and numerically.

All three approaches result in both pointwise and pathwise smoothing compared to the target, but reducing the drift yields better pointwise smoothing for similar levels of pathwise smoothing.

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