Estimation and Risk Modelling under Parity Principles

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Abstract

Risk parity/budgeting is a concept introduced in mid 1990's by asset management practitioners aiming to create a robust investment strategy that is less sensitive to the changes in market risk. We provide a mathematical characterization for risk parity/budgeting portfolio construction problems under general risk preferences. For the general problem when distribution of returns is not known, we demonstrate the existence of a solution to the risk budgeting problem for any convex and homogeneous risk preferences. Statistical inferences are determined for those portfolios when risk preferences are ordered by variance or Conditional Value-at-Risk. A novel Conditional Value-at-Risk estimator is proposed, which is shown to perform very well on non i.i.d observations, based on simulated and real-life data, especially during periods of bull market and irrational exuberance. Our numerical results show superior performance of risk parity portfolios in terms of various measure of performance such as Sharpe ratio and diversification when comparing with other benchmark portfolios including the equally weighted portfolio. These results are detailed in [1].

We then show how *parity principles* could help to conceptualize a new estimation methods based on homogeneous functionals. As a proof of concept, we illustrate our point by showing the deployment of parity estimation on Multiple Linear Regression, and we name this as *Parity Regression*; further details could be found in [2]. Similar deployments one could achieve for

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other machine learning and statistical models such as neural networks with certain activation functions and generalised linear models with certain link functions.

Multiple Linear Regression is one of the simplest estimation method that exists in the machine learning and statistical fields that have been adopted by actuarial science, natural sciences, social sciences, and any other fields that rely on machine learning and statistical tools, which makes Parity Regression a good candidate to explain the new concept coined as *Parity Estimation*. We first describe Parity Estimation that could be applied for any homogeneous risk minimization objective functions. Since the standard risk minimization problem is a special case of our Parity Estimation methodology, we believe that such machine learning and statistical methodologies would benefit from our novel estimation methodology. A short mathematical characterization of our methodology is provided, and explain how existing algorithms could be used to solve our proposed estimation method. Finally, our numerical examples show the advantages of using our new estimation method as compared to its linear regression competitors such as *Ordinary Least Estimator* and *Ridge Regression Estimator*.

Keywords: Risk budgeting/parity; Parity Estimation; Multiple Linear Regression.

References

[1] V. Asimit, L. Peng, R. Tunaru and F. Zhou (2024), "Constructing optimal portfolio under risk budgeting." *Preprint*, https://tinyurl.com/ys76cpvu.

[2] V. Asimit, A. Badescu, Z. Chen and B. Ichim (2024), "Parity Regression." *Working paper*.