A Mean-Field Extension of the LIBOR Market Model for the Valuation of Long-Term Guarantees

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Abstract

We introduce a mean-field extension of the LIBOR market model (LMM) which preserves the basic features of the original model. This mean-field LIBOR market model (MF-LMM) is designed to reduce the probability of exploding scenarios, arising in particular in the market-consistent Monte Carlo valuation of long-term guarantees. This is a practically relevant extension as LIBOR market models (LMMs), respectively their suitors, are nowadays widely used by practitioners to valuate market instruments which depend on interest rate movements such as caps or swaptions. However, LMMs have been developed with a view towards valuating interest rate derivatives with a maturity/tenor structure at the order of months or a few years. Recently, the valuation of long-term guarantees has become increasingly important in the life insurance sector. The regulatory framework Solvency II (c.f. [1]), which was implemented by the European Union per January 1, 2016, requires European insurers to assign market consistent values to their liability portfolios. However, because of the projection horizon of a typical life insurance portfolio, the generated scenarios often suffer from blow-up. In this context, blow-up or explosion means that there are a significant number of scenarios (e.g. more than 1%) such that the forward rate (for any maturity and any point in time) exceeds a predefined threshold (e.g. 50% of interest). This is referred to as the explosion problem of LMMs which is also theoretically well-known.

In order to resolve this problem, this paper provides the following contributions:

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- The MF-LMM is introduced and its existence is proved by showing that the corresponding mean-field SDE is well-posed and has a unique strong solution. This model is capable of significantly reducing the probabilty of exploding paths by adding a dampening factor to the classical LMM, which depends on the variances of the LIBOR rates themselves. Such a dampening factor that depends on the process itself, turns the classical underlying SDE into a mean-field SDE.
- We then address practical aspects of the MF-LMM such as a Black '76-type formula for a given measure flow, calibration and change to the spot measure.
- We demonstrate in a numerical study that the model can be used to effectively reduce explosion. The corresponding Monte Carlo method is based on a suitable interacting particle system which approximates the underlying mean-field equation. Furthermore, the effects on cap and swaption prices are studied, and it is shown that these can be (approximately) preserved by a judicial choice of the mean-field component.
- We also address the theoretical aspects such as the existence and uniqueness of the solution to the underlying mean-field SDE.

The talk is based on [2].

Keywords: LIBOR market models; solvency II; valuation of long-term guarantees; life insurance portfolios; exploding rates; mean-field games; Black formula.

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References

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