Enhancing SME Factoring: A Stackelberg Game-Based Hybrid Pricing Model

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Abstract

In supply chain management and trade-credit, buyers of goods or services are often granted a delayed payment goal, and the sellers of the respective goods or services are thus exposed to credit risk. Factoring is a financing decision by which sellers can eliminate that risk from their balance sheet. This service is typically offered on a whole turnover basis, however, SMEs prefer financing on a single debt level, and, hence, adverse selection emerges as an additional source of risk. This working paper consists of three main chapters, namely the development of a multivariate default model, an optimization problem for the seller and the derivation of a hybrid linear pricing model that covers both default and adverse selection risk. First, as the trade-credit period we consider is typically not longer than 90 days, we assume a tractable one-parametric model for the univariate default times: the exponential distribution. Hence, we let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space supporting $d \in \mathbb{N}$ random variables $\tau_k, k \in \{1, \dots, d\}$, which we define by means of their survival function

$$\mathbb{P}(\tau_k > t) = e^{-\Lambda_k t}, \ \forall k \in \{1, \dots, d\}, \ \Lambda_k > 0.$$

Extending the exponential law from the perspective of its lack-of-memory property to a multivariate analogon leads to the *Marshall–Olkin distribution*, first derived in [1]. Overcoming the implied complexity of the parameters, researchers have developed and derived a connection between the (extendible) Marshall–Olkin distribution and the notion of Lévy subordinators. Using this connection, we let the portfolio-default model consist of $n \in \mathbb{N}_{>0}$ risk factors with $m_n \in \mathbb{N}_{>0}$ categories, respectively. This way, we can define the default time for each company $k \in \{1, \dots, d\}$ as the first-passage time of a Lévy subordinator across a unit exponential threshold:

$$\tau_k = \inf\{t > 0 : L_t^{(k)} > \varepsilon_k\}.$$

After setting up the portfolio default model, which allows us to efficiently simulate the set of companies defaulting in $[0, t]$, we set up a portfolio optimization problem for the seller. This approach is part of the class of bi-level optimization problems ([2]) as we are trying to find a

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price model for the factoring company by optimizing their opponents decision. After defining the objective function, we state and solve the following optimization problem:

$$\begin{aligned} \min_{\mathbf{x}} \quad & g_\alpha^*(\mathbf{x}) \\ \text{s.t.} \quad & \sum_{k=1}^{d} x_k M_k \leq P^*, \\ & \mathbf{x} \in [0,1]^d. \end{aligned}$$

We show strong convexity of this function and argue why it is necessary to use a stochastic gradient descent algorithm to solve this problem. We conclude with almost sure convergence of the underlying algorithm and derive conditions for both seller and factoring company to enter into an agreement, namely covering expected losses and paying less than what is potentially earned. This implies certain boundaries for the price, i.e. a price interval. Next, we assume, that in case of a default, a factoring company's recovery value is higher than the one from the seller (follows as the seller is assumed to be a SME). Then, we show that the price interval given by

$$f_k := \mathbb{E}\left[\left(1-\kappa_k^{fac}\right)\mathbb{1}_{\{k\in S_t\}}\right]\left(1+\psi\right),$$

for debtor $k \in \{1,\dots,d\}$ satisfies the before derived interval conditions and enables the possibility to include a dynamic part in the price. κ_k^{fac} represents the recovery value of the factoring company, S_t the set of defaulted companies in $[0,t]$ and $\psi = c + m + \varphi$ is a combination of management costs, margin and the dynamic part φ. Lastly, we embed the simplest data-rich modeling approach for φ in our analysis, namely a linear dynamic pricing model with demand covariates [3]. In summary, we derive a lower and an upper bound for the price and then show that it suffices to deploy a linear model as a mix of a static and a dynamic part to cover both default and adverse selection risk. This way, factoring companies can offer coverage on a single debt level without underestimating the risk. Munich Re, the largest reinsurer in the world has sponsored this project which resembles the interest of the industry in this field.

Keywords: Stackelberg game, trade-credit insurance, SME factoring, dynamic pricing, hybrid pricing, Lévy-frailty model, portfolio optimization.

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