The valuation and assessment of retirement income products: A unified Markov chain Monte Carlo framework

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Abstract

Pension savings and residential assets are the key asset classes for retirees in the developed markets. Due to the lack of clear drawdown strategies for pension savings, retirees are asset-rich and income poor in the post-retirement phase. As a way of encouraging retirees to drawdown their savings, it is imperative effective awareness tools highlighting features of various retirement income products.

This paper predominantly focuses on the Australian context and devises a flexible assessment framework for a catalogue of existing retirement income products which include, account-based pensions, group self-annuities and variable annuity contracts. Graphical illustrations for the risk-return trade-offs for each product are presented which can readily be adapted by advisors, and all stakeholders as a tool for enhancing the decision-making process for retirees. The key features of the retirement income products which include variable annuities(VAs)[1], account-based pension, and group self-annuities (GSAs), are presented in an easily understandable way. We also conduct a sensitivity analysis on the investment options of the underlying fund to provide insights for retirees to maximise income according to their risk preferences. The proposed assessment framework utilises the Hamiltonian Monte Carlo (HMC) algorithm for valuation of retirement income products.

A common feature of the aforementioned products is that the income benefits are linked to an underlying investment fund. Pricing of income products involves evaluating the expectation of future benefits and fund values. Typically, a pension fund consists of multiple asset classes[2]. As

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[1] In this research, we consider VAs embedded with guaranteed minimum living benefit (GMLB) including guaranteed minimum maturity benefit (GMMB), guaranteed minimum income benefit (GMIB) and guaranteed minimum withdrawal benefit (GMWB). For the living benefit riders considered in this paper, we explore cases with and without spouse benefit options.

[2] For example, the MultiOption Guide B Series Variable Annuity issued by Minnesota Life offers various investment fund choices to the policyholders including the BlackRock International Index V.I. Fund, whose investments are allocated to stocks in more than ten countries.

the number of assets in the fund increases, the concentration of measure[3] makes the valuation of retirement income products a challenging task due to the high-dimensional nature of the resulting framework [2]. We devise the Hamiltonian Monte Carlo (HMC) simulation to overcome this difficulty. The HMC method originates from quantum physics [1] and it is widely applied in high-dimensional simulation and statistics. The HMC algorithm makes Markov chain transitions according to the Hamiltonian dynamics to reduce the sample autocorrelation in order to obtain a fast exploration of the typical set. We provide a closed-form solution to the HMC dynamics and it greatly improves the algorithm's efficiency.

Our numerical results highlight that in higher dimensions, the Hamiltonian Monte Carlo algorithm is computationally efficient in generating low-correlated investment return samples such that it is relatively more accurate than the crude Monte Carlo in terms of numerical integration. Key findings of this paper are as follows, (i) GSA products generate stable and smoothly increasing income due of dynamic pooling, and they best protect retirees from longevity risk in generating a high consumption profile, as there is no bequest component; (ii) VAs generate the most stable living benefits and the account-based pension provides the highest but the most volatile living benefits at older ages; (iii) GMWB with the spouse benefit option yields the highest lifetime utilities among the products.

Keywords: Retirement income products; Hamiltonian Monte Carlo (HMC); Account-based pension; Variable annuities (VAs); Group self-annuities (GSAs).

References

[1] Alder, B.J. and Wainwright, T.E., (1959), "Studies in molecular dynamics. I. General method. " *Journal of Chemical Physics*, 31. pp. 459-466.

[2] Betancourt, M., (2017), "A conceptual introduction to Hamiltonian Monte Carlo. " *arXiv,* arXiv:1701.02434.

[3] As dimension increases, the volume of the random variables away from the mode increases exponentially, but the density becomes negligible further away from the mode. The tension between the increasing volume and decreasing density mass increases rapidly in high dimensions. For a random variable in high dimensions, its typical set becomes singular and much smaller than the total volume, which makes the traditional Monte Carlo and other quadrature techniques inefficient in high-dimensional integration.