A Discrete-Time Hedging Framework for Econometric Option Pricing Models

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Extended Abstract

We present a hedging framework for a general class of discrete-time affine multi-factor processes that encompasses most of the option pricing models proposed in the econometric literature. In particular, our framework accommodates models with multi-component volatility factors, fat tails, leverage effects, and a non-monotonic pricing kernel.

This article makes two important contributions to the literature: one theoretical and one empirical. *First*, at the theoretical level, we derive a semi-closed form expression for a risk-minimization hedging strategy under a general class of discrete-time affine multi-factor models. Although option pricing formulas have been derived for various discrete-time affine models and are now well-known, hedging expressions are not yet available for most of the models nested in our general affine framework. Our hedging formula is expressed as an inverse Laplace (or Fourier) transform and takes advantage of the fact that many European derivatives, such as calls and puts, have payoff functions that admit an inverse Laplace transform representation (see Hubalek et al., 2006). Our formula applies to many econometric option pricing models proposed in the literature such as the multi-component fat-tailed GARCH model considered by Babaoğlu et al. (2018), the Lévy GARCH model of Ornthanalai (2014), the generalized affine realized volatility (GARV) model of Christoffersen et al. (2014), and the heterogeneous autoregressive gamma (HARG) model for realized volatility of Corsi et al. (2013) as well as its extensions considered by Majewski et al. (2015), Alitab et al. (2020), and Bormetti et al. (2020).

Second, at the empirical level, we conduct an extensive experiment with option data, namely 666,790 S&P 500 Index option contracts (henceforth SPX options) covering the period 1996–2018, to determine the extent to which multi-component volatility factors, fat tails, and a non-monotonic pricing kernel can improve hedging performance. To assess the impact of modelling features on hedging effectiveness, we consider eight affine GARCH models that combine (i) one or two volatility components, (ii) Gaussian or inverse Gaussian (IG) innovations, and (iii) a monotonic or non-monotonic pricing kernel. Every option is hedged by dynamically trading in the underlying S&P 500 Index on a daily rebalancing frequency and until its maturity date. Our study is the first to examine which of these features are indispensable to effectively hedge options. We remark that the ability to compute optimal hedging ratios in a split second is key to make our empirical hedging experiment feasible from a computational standpoint. Overall, we find that each one

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of the three model features leads to a statistically significant reduction in the root-mean-square error (RMSE) of terminal hedging errors on the basis of a Diebold and Mariano (1995) test. Fat tails can be credited for half of the hedging improvement observed, while a second volatility factor and a non-monotonic pricing kernel each contribute to a quarter of this improvement. In addition, we find that longer maturity options and options issued in the last five years of our data sample benefited the most from richer model dynamics. Moreover, our study indicates that our three modelling features impact pricing and hedging performance differently. A robustness analysis verifies that a similar conclusion can be reached when considering DJIA options, although the contribution of each feature is no longer the same as for SPX options. Finally, we perform an additional robustness exercise to explore whether the use of a hedging-based loss function in the parameter estimation process can enhance the out-of-sample hedging performance of the models considered in our study. This exercise suggests that the choice between a pricing or hedging-based loss function in the estimation process has a rather marginal impact on performance.

Keywords: option hedging, risk-minimization, affine models, multi-component volatility, exponential-affine pricing kernels.

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