On the Distribution of VaR Exceedances in a Vt-s-vine Model

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Abstract

In the banking and insurance industries, probability distributions are used to model and quantify risk. Given a profit and loss (P&L) distribution, risk measure provides a point estimate of the amount of capital required as a cushion against insolvency. The main risk measure used by financial institutions and regulators is Value-at-Risk (VaR), which is defined as a quantile of the P&L distribution. The Basel regulations for market risk at banks assume that if a bank has an adequate risk model, the number of VaR exceedances is binomially distributed with n being the number of days backtesting and p is the VaR percentile (1% typically for 99% VaR). Under this assumption, the Basel committee on Banking Supervision (2019) applies the traffic light system as a regulation based on a sample of $n = 250$ observations to define the backtesting green, amber and red zones according to the critical number of exceedances of levels based on 99% VaR.

It is worth noting that the quantile exceedances indicators variables are supposed to be independent in Basel regulations. However, the quantile exceedances indicators variables tend to be positively dependent in practice. The serial dependence in the P&L cased by stochastic volatility should be taken into consideration. Therefore, the distribution of exceedances in the P&L is not a binomial distribution and it will tend to have a heavier tail and a higher variance.

There are three main questions studied in this paper. Firstly, we investigate the difference between the distributions of the quantile exceedances of P&L in the independent and identically distributed (iid) case (in Basel regulations) and the non-iid case (in practice). We use simulations of vt-s-vine models to mimic the serial dependence in the P&Ls and explore the influence of the dependence on the distribution of quantile exceedances. The vt-s-vine models consist of v-transform (details can be found in McNeil (2021)) and s-vine copulas (details can be found in Bladt and McNeil (2022)). In order to simplify the simulation processes, we apply the linear v-transform in this paper. The exceedances probabilities are calculated and simulated in iid and non-iid case, respectively. Secondly, we employ the six widely used empirical quantile estimator approaches, which are referred to in Hyndman and Fan (1996), to explore which method banks should prefer in the sample with dependence described by vt-s-vine models. Finally, we model the quantile exceedances in P&Ls and apply the beta-binomial distribution to approximate the

quantile exceedances of simulations from vt-s-vine models. The distribution has great influence on setting the levels of the three backtesting zones in the traffic light system, because the binomial distribution can not be used in non-iid case.

The first point we find is that the accuracy of empirical quantile estimates of VaR decreases when the losses have serial dependence. Moreover, the choice of bivariate copulas in vt-s-vine models affects the selection of quantile estimate methods. The empirical quantile estimates of VaR by the six approaches are worse for non-Gaussian copulas, compared to Gaussian copulas. Generally, the third method usually overstates VaR and yields the best exceedances estimation in both iid and vt-s-vine cases, which reveals it is a conservative approach. Another counterintuitive finding is that the quantile estimator is closest on average to the true value of the quantile of a distribution may not be the one gives the most accurate value for exceedance probability. Since regulation is based on numbers of exceedances and not closeness to the unknown true value of VaR, this gives an indicator as to the method of empirical quantile estimation that banks should prefer. Finally, it is surprisingly to find that the exceedances from vt-s-vine with Gaussian copula seqeunces can be modelled so accurately by beta-binomial distributions. The exceedances of processes with non-Gaussian copula sequences can be modelled by beta-binomial distribution as well, but it depends on the selection of quantiles and sample sizes.

Keywords: Value-at-Risk; Exceedances; Vt-s-vine; V-transform; S-vine copulas; Beta-binomial distribution.

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