

# Market Price of Longevity Risk for A Multi-Cohort Mortality Model with Application to Longevity Bond Option Pricing

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## Abstract

The pricing of longevity-linked securities depends not only on the stochastic process for the underlying risk factors, but also the attitude of investor towards the risk of those factors. The Life & Longevity Markets Association (LLMA) has identified the market risk premium of longevity risk as one of the key inputs in a longevity pricing framework<sup>1</sup>. To determine the market risk premium, a common practice is to use available market prices, such as longevity-linked securities and longevity indexes. The longevity market is however incomplete due to the lack of traded assets, and calibration of market prices of risk is a challenge. The risk-neutral pricing measure cannot be uniquely determined. [7] propose to use a Wang transform for the securitization of longevity risk, and the market price is defined as the shift parameter in the Wang transform to risk adjust a survival distribution based on life tables and annuity quotes. A different approach is taken in [2] and [4] who propose to derive parameter values for stochastic mortality models using survival probabilities implied by annuity prices, so that the market price of longevity risk is implicitly included in these parameter values. The Wang transform has limitations in pricing longevity risk [1]. There remains no well accepted method to calibrate and incorporate the market price of longevity risk into mortality dynamics under a risk-neutral measure for market valuation that can be used in the valuation of longevity-linked products.

In this research, we investigate how to derive the market price of longevity risk using investable retirement indexes. The BlackRock CoRI Retirement Indexes which provides a daily level of the estimated cost of lifetime retirement income for 20 cohorts in the U.S. are adopted for this purpose. We utilise a multi-factor mortality modelling framework based on affine processes which has been used extensively in [6, 5, 3, 8]. The benefits of affine processes are that they are analytically tractable and ease of application to valuation and risk management of longevity linked product and financial instruments.

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<sup>1</sup>LLMA(2010). Longevity Pricing Framework. [www.llma.org]

When calibrating the market prices of mortality risk to the CoRI Indexes which consist of 20 cohorts, we assume that the risk premiums for common factors are the same across cohorts, but the risk premiums for cohort specific factors are allowed to vary. The market prices of longevity risk are then calibrated by matching theoretical risk-neutral index prices with BlackRock CoRI index values. Closed-form expressions and prices for European options on longevity zero-coupon bonds are derived using the model and the calibrated market prices of longevity risk. We show that call prices on longevity bonds are increasing functions of the option maturity. However, with an increase in bond maturity, call prices increase first and decrease thereafter. Furthermore, we compare the prices of longevity zero-coupon bond options with the prices of zero-coupon bond options. For shorter bond maturities zero-coupon bond options are cheaper while for longer maturities longevity linked zero-coupon bond options become cheaper.

**Keywords:** Multi-cohort mortality model; Market price of longevity risk; Longevity indexes.

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