

# Time-Consistent Strategy for a Multi-Period Mean-Variance Asset-Liability Management Problem with Stochastic Interest Rate

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## Abstract

Asset-liability management (ALM), also known as surplus management, is concerned with the difference between the asset value and the liability. It is an important concern not only for investment institutions but also for individual investors. Thus, ALM problem is of both theoretical interest and practical importance, has received considerable attention for the last decade in the actuarial and the financial literature, and is the topic of this paper.

Markowitz's mean-variance model is an important criterion for modelling ALM problems. In the mean-variance model, the investor aims to determine the optimal investment strategy that maximize the expected terminal wealth and to minimize the risk measured by the variance of her terminal wealth. Due to the non-separability of the variance operator, the Bellman optimality principle can not be directly applied to dynamic mean-variance models, and thus the optimal investment strategies of these models do not satisfy the time consistency. In literature, such optimal portfolio strategies are called pre-commitment strategies. A pre-commitment strategy is just optimal from the viewpoint of the initial time and decision-makers at any future time  $k$  must commit themselves to following the initial optimal strategy even if it is not optimal at time  $k$ . Thus, the pre-commitment strategy is time-inconsistent, which has been criticized for lacking rationality. For example, since the investment psychology and tastes will often change over time, the rational investor at a later time can not commit to following a strategy that is not optimal at that time. Hence, getting time-consistent strategies is more important. However, Existing research on time-consistent strategies for ALM problems is very little and is limited to the framework of continuous-time. To the best of our knowledge, time-consistent strategies for ALM problems under the multi-period mean-variance framework have not been studied in the literature.

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On the other hand, in our real life, the interest rate does change inevitably sometimes. Moreover, marketization of interest rate results in more frequent changes in interest rates. Hence, a consensus has been formed for quite a long time to model the interest rate as a stochastic process, and, of course, to consider a stochastic interest rate as one of the important uncertain factors in investment. Commonly used stochastic interest rate models are Vasicek model, Cox-Ingersoll-Ross (CIR) model and Ho-Lee model. Some scholars have investigated the portfolio problems and ALM problems with stochastic interest rate. But their works are limited to continuous-time case. In practice, however, the investment strategy can only be implemented in a discrete-time manner, since investors can only rebalance their positions from time to time. To the best of our knowledge, time-consistent strategies for multi-period mean-variance portfolio selection problems with stochastic interest rate have not been studied in the literature, not to mention a multi-period mean-variance ALM problem with stochastic interest rate.

With the above in mind, the purpose of this work is to study the time-consistent strategy for an ALM problem with stochastic interest rate under a multi-period mean-variance framework. We use the discrete-time Vasicek stochastic interest rate model proposed by Yao et al. (2016) to describe the stochastic process of interest rate. Similar to Björk and Murgoci (2014), we regard this problem as a non-cooperative game whose equilibrium strategy is the desired time-consistent strategy, and derive the analytical expressions of the equilibrium strategy, the equilibrium value function and the equilibrium efficient frontier by a system of extended Bellman equations. In addition, we discuss some special cases of our model, and give some properties of our equilibrium strategy, especially a multi-period version of the two-fund separation theorem. Finally, we use real data to analyze the impact of the liability and the stochastic interest rate on the equilibrium strategy and the efficient frontier.

The main contributions of this paper are as follows. (i) The time-consistent strategy of a multi-period mean-variance ALM problem is studied for the first time. (ii) We are also the first to study the time-consistent strategy for the multi-period mean-variance investment problem with stochastic interest rate. (iii) We obtain the equilibrium strategy and the corresponding efficient frontier for our problem in closed-form. (iv) We use the matrix technology to derive out results, which may shed light on the research of the relevant dynamic optimization problems.

**Keywords:** Multi-period mean-variance model, Time-consistent strategy, Asset-liability management, Stochastic interest rate, Game theory.

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