

An Approximation Method for Risk Aggregations and Capital Allocation Rules Based on Additive Risk Factor Models

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Abstract

Risk aggregation is a pervasive issue in finance and insurance. In the context of additive risk factor models, the quantities of interest related to aggregation are determined by individual risk factors and interdependence within the summation. In the majority of cases, the aggregation is influenced by one or more common factors—such as geography, inflation or economic environment—as well as certain idiosyncratic characteristics. Due to such multivariate complexity, the joint distribution of the summands is usually out of reach and the probability distribution of the aggregation is either too difficult to specify or cumbersome to work with. Consequently, abundant literature delves into finding accurate approximations that are practical and tractable to compute the quantities of interests. One successful approximation is the aggregation’s lower bounds in the sense of convex order. In particular, the so-called “maximal variance” convex lower bound has been shown both precise and tractable, especially for the sum of follow lognormal distributed random variables.

In this paper, we develop the convex lower bound approximation method for risk aggregations in the context of generalized Gamma distribution. Such method is preferable due to its tractability and analytical results, which facilitates straightforward computation of quantities and analysis of interest. We provide explicit

solutions under the additive risk factor models based on this method. In particular, we work out an approximate CTE-based capital allocation rule. As another distinguishing contribution, we further consider a model with contingent risk factors and extend our method to this model. By doing so, we significantly extend the applicability of our method. We show that the approximation convex lower bound method is still valid despite of the additional randomness and complexity. Moreover, we conduct stress test to show that the approximate CTE capital allocation rule is very robust with respect with respect to various levels of the joint default rates of the model.

Keywords risk aggregation, convex order, capital allocation, lower bound approximation.