

Approximations of the Gerber-Shiu function for risk models with two-sided jumps

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Abstract

In this paper, we study the Gerber-Shiu function of a two-sided risk model, where the negative jumps describe claims and the positive ones describe the stochastic income. In particular, we assume that the positive jumps are of phase-type, while the negative jumps are heavy-tailed. This model recasts with the aid of fluid embedding as a spectrally negative Markov Additive Process (MAP), for which occupation densities and the scale matrix need to be evaluated [2]. Note that closed-form expressions for the Gerber-Shiu function are available when the negative claims are also of phase-type [1]. Therefore, by relating our model to a risk model with two-sided phase-type jumps, our aim is to derive accurate approximations for the Gerber-Shiu function. More precisely, we derive a ‘matrix’-expansion of the Gerber-Shiu function via utilising perturbation analysis alongside with results of spectrally negative Lévy processes, and from this construct our approximations, which maintain the computational tractability of phase-type approximations and provide small errors.

Keywords: Gerber-Shiu function, spectrally negative MAPs, phase-type, heavy-tails, approximations, perturbation analysis

References

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