

# Survival Analysis of Pension Scheme Mortality With Missing Data

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## Abstract

The risk management of a portfolio of exposures needs statistical models allowing for the inclusion of the main risk factors. This study deals with pension scheme datasets where the future lifetime of each individual is modelled by a parametric model for the force of mortality. In particular, we specify a functional form for the hazard function which depends on individual covariates and unknown parameters. The latter are estimated by using likelihood-based techniques. We extend the work of [1] and [2] as we address the inferential problem where some covariates are missing for part of the pension scheme mortality experience: in this case the model should account also for the distribution of the missing covariates, which are categorical and thus multinomially distributed. We analyse parameter identifiability and propose an algorithm to handle the estimation task, transforming a constrained optimization problem into an unconstrained one, and then using a Newton-Raphson iterative routine. This leads to less parameter uncertainty than discarding observations with missing components. Furthermore, we devise a bootstrap approach for the estimation of the variance-covariance matrix of parameters estimates. Finally, we analyse the financial impact due to the inclusion of the covariates, and due to the exclusion from the fitting process of part of the mortality experience where data are missing, particularly on the annuity factor and on the mis-estimation risk capital requirement.

**Keywords:** Mortality, Survival model, Longevity Risk, Missing data, Mortality models with covariates

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## References

- [1] Richards S.J. (2008), “Applying Survival Models to Pensioner Mortality Data.” *British Actuarial Journal*, vol. **14**(2), pp. 257-303.
- [2] Richards S.J., Kaufhold K., Rosenbusch S. (2013), “*Creating portfolio-specific mortality tables: a case study.*” *European Actuarial Journal*, vol. **3**, pp. 295-319.