Stochastic Stackelberg differential reinsurance games

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Abstract

We consider a new continuous-time framework to analyze optimal reinsurance, in which an insurer and a reinsurer are two players of a stochastic Stackelberg differential game, i.e., a stochastic leader-follower differential game. This allows us to determine optimal reinsurance from joint interests of the insurer and the reinsurer, which is rarely considered in a continuous-time setting. In the Stackelberg game, the reinsurer moves first and the insurer moves subsequently to achieve a Stackelberg equilibrium towards optimal reinsurance arrangement. Speaking more precisely, the reinsurer is the leader of the game and decides on optimal reinsurance premium to charge, while the insurer is the follower of the game and chooses optimal proportional reinsurance to purchase. We solve the game problem in two cases: exponential utility maximization and mean-variance optimization. Under the utility maximization framework, we find that the reinsurer always applies the variance premium principle to calculate the optimal reinsurance premium and the insurer's optimal ceding/retained proportion of insurance risk depends not only on the risk aversion of itself but also on that of the reinsurer. Under the mean-variance framework, if the reinsurer adopts the variance premium principle (resp. expected value premium principle), then the Stackelberg equilibrium is attained when the insurer purchases the proportional reinsurance (resp. excess-of-loss reinsurance).

Keywords: Stackelberg game; Proportional reinsurance; Excess-of-loss reinsurance; Backward stochastic differential equation; Variance premium principle; Expected value premium principle

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