

# A Branching Process Approach to Power Markets

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## Abstract

Energy markets, and in particular, electricity markets, exhibit very peculiar features. The historical series of both futures and spot prices include seasonality, mean-reversion, spikes and small fluctuations.

After the pioneering paper by Schwartz, where an Ornstein-Uhlenbeck dynamics is assumed to describe the spot price behavior, several different approaches have been investigated in order to describe the price evolution. A comprehensive presentation of the literature until 2008 is offered in the book by F.E. Benth, J. Salthythe-Benth and S. Koekebakker [4].

High frequency trading, on the other hand, introduced some new features in commodity prices dynamics: in the paper by Filimonov, Bicchetti, Maystre and Sornette [5] evidence is shown of endogeneity and structural regime shift, and in order to quantify this level the branching ratio is adopted as a measure of this endogenous impact and a Hawkes processes dynamics is assumed as a reasonable modeling framework taking into account the self-exciting properties.

The purpose of the present paper is to propose a new modeling framework including all the above mentioned features, still keeping a high level of tractability. The model considered allows to obtain the most common derivatives prices in closed or semi-closed form. Here with semi-closed we mean that the Laplace transform of the derivative price admits an explicit expression.

The models we are going to introduce can describe the prices dynamics in two different forms, that can be proved to be equivalent: the first is a representation based on random fields, the second is based on Continuous State Branching Processes with Immigration (CBI in the following). The idea of adopting a random fields framework for power prices description is not new: O.E. Barndorff-Nielsen, F.E. Benth and A. Veraart introduced the Ambit Fields to

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this end, showing how this approach can provide a very flexible and still tractable setting for derivatives pricing [1], [2]. Moreover CBI processes exhibit the Markov property, while Hawkes processes enjoy this property only in particular cases, and this feature makes our approach more appealing.

A model based on CBI has been proposed recently by Y. Jiao, C. Ma and S. Scotti in view of short interest rate modeling, and in that paper it was shown that, with a suitable choice of the Lévy process driving the CBI dynamics, the model can offer a significant extension of the popular CIR model [6].

The model we propose extends in different ways some relevant models already available in the literature. It belongs to the class of arithmetic models (following the classification proposed by F.E. Benth, J. Salhyte-Benth and S. Koekebakker), and the driving processes are Lévy processes with positive jumps, i.e. subordinators, so it extends the model proposed by F.E. Benth, J. Kallsen and T. Meyer-Brandis [3] by formulating the dynamics via a random field approach, which allows to include some self-exciting features. On the other hand, the random field approach highlights some similarities with the Ambit Field-based models introduced by O.E. Barndorff-Nielsen, F.E. Benth and A. Veraart [2].

**Keywords: Branching Processes, Energy Markets, Self-Exciting Structures, Risk Premium Term Structure, Random Fields.**

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