Capital Injections and Dividends with Tax

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Abstract

The surplus of an insurance portfolio is modelled by a spectrally negative Lévy process $\{X_t^0\}$. We denote the initial capital by $x = X_0^0$. Dividends may be paid and, if the capital becomes negative, capital injections have to be made in order to keep the surplus positive. Using a dividend strategy $\{D_t\}$ — an increasing process with $D_{0-} = 0$ — the surplus is $X_t^D = X_t^0 - D_t + L_t$, where the capital injections $\{L_t\}$ is the smallest increasing process such that $X_t^D \geq 0$ for all t.

In addition tax has to be paid for dividends, but capital injections lead to exemptions from tax. Let $1-\gamma$ be the tax rate, where $\gamma \in (0,1)$. We denote by Y_t the maximal amount of dividends that could be paid without tax. Letting $Y_0 = y$, we have

$$Y_t = y + L_t - \int_0^t \mathbb{1}_{Y_s>0} \, \mathrm{d}D_s^c - \sum_{s\leq t} \min\{\Delta D_s, Y_{s-}\} \;,$$

where $D_t^c = D_t - \sum_{s\leq t} \Delta D_s$ is the continuous part of the dividend payments D_t. The value of a dividend strategy $\{D_t\}$ is

$$\begin{aligned} V^D(x,y) \;\;=\;\; & \mathbb{E}\Bigl[\int_0^\infty \mathrm{e}^{-\delta t}(\mathbb{1}_{Y_t>0} + \gamma \mathbb{1}_{Y_t=0}) \, \mathrm{d}D_t^c \\ & + \sum_{t\geq 0} \mathrm{e}^{-\delta t}[\min\{\Delta D_t, Y_{t-}\} + \gamma(\Delta D_t - Y_{t-})^+] - \eta \int_0^\infty \mathrm{e}^{-\delta t} \, \mathrm{d}L_t\Bigr] \;, \end{aligned}$$

where $\delta > 0$ is a preference parameter and $\eta \geq 1$. The value function is then $V(x,y) = \sup_D V^D(x,y)$. Under the optimal strategy an alternative interpretation of the tax is that the company has to pay tax whenever the surplus without dividends $\{X_t^0\}$ is at a maximum, so that the income of the portfolio is diminished and less tax is paid.

Let $V^1(x)$ be the value function of the problem without tax, that is $\gamma = 1$. In the latter case, the optimal strategy is a barrier strategy with a barrier at b_1. We show that $V(x,y) = V^1(x) - C\mathrm{e}^{\rho(x-y)}$ for $x \leq b_\gamma$ if $y = 0$ and for $x \leq b_1$ if $y > 0$, where ρ is the positive solution to

$$\mathbb{E}[\exp\{r(X_t^0 - x)\}] = \exp\{t\delta\} \;,$$

and C is chosen such that $\inf_x \frac{\partial}{\partial x} V(x,0) = \gamma$. The barrier b_γ is the point $\frac{\partial}{\partial x} V(b_\gamma, 0) = \gamma$. The optimal strategy is then a two barrier strategy: if $Y_t > 0$, pay all capital above b_1 as dividend. If $Y_t = 0$, pay all capital above b_γ as dividend. Examples are the diffusion case treated in [2], the classical risk model treated in [1], or the perturbed risk model treated in [3].

Keywords: Lévy risk model; dividends; capital injections; tax; barrier strategy; Hamilton–Jacobi–Bellman equation; perturbed risk model.

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