

# Inefficient market bubbles

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## Abstract

Following the understanding that asset price bubbles are generated by market failures, we present a framework for explosive semimartingales that is based on the antagonistic combination of (i) an excessive pre-crash process and (ii) the random time of a drawdown. We show that "rational expectation bubbles" are by design afflicted with an inherent error in both continuous ("strict local martingale") and discrete time models.

Our framework significantly extends the range of feasible asset price processes during times of excessive growth. It will simplify and foster interdisciplinary exchange at the intersection of economics and mathematical finance and encourage further research.

## Details

Asset price bubble models can essentially be grouped into two types – those where bubbles appear in a fully efficient market and those that are based on a violation of market efficiency.

**Rational expectations bubbles** The first type has become known as *rational expectation bubbles* and is characterized by a rationally determined fundamental value  $S^*$ ,

$$S_t^* = \mathbb{E}_{\mathbb{Q}}[(D_{\infty} - D_t) + S_{\infty} | \mathcal{F}_t] \quad (1)$$

for cumulative dividend payments  $D$  and a final payoff  $S_{\infty}$ . We argue that this definition of the fundamental value leads to joint hypothesis problems in both discrete (see, e.g., [1] for an overview) and continuous time models ([2], [3]). While the discrete time case has been extensively discussed in the literature and is most criticized for a structure that is based on a payoff at infinity, in continuous, finite time the problems rests on an ambiguity of the fundamental value.

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**Inefficient market bubbles** The second type has seen a flurry of diverse mechanisms trying to explain and model a break of *perfect information rational expectations* – such as investor irrationality, ambiguous information, certain limits to arbitrage ([4] for a concise overview) or positive feedback activity ([5], [6]). The present work is an attempt to provide a model framework for this second type of processes and is based on a nonlinear decomposition of the asset price into

1. a pre-crash process  $\tilde{S}$  (that may be explosive),
2. a random time of the crash  $\tau_J$  (the “Minsky moment”) and
3. the shape  $X$  of the crash.

Instead of a linear decomposition in fundamental value and bubble value,  $S = S^* + B$ , as usual in the rational expectations framework, our decomposition complies with basic stylized facts of a bubble and allows one to transcend the rational expectations framework and use explosive (or inefficient) pre-crash processes  $\tilde{S}$ .

**Keywords:** Financial Bubbles, Explosive processes, Bubble decomposition, Strict local martingale approach, Infinite horizon bubbles.

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