Parameter Uncertainty with a Hypermodel and Fat-tailed Innovations

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Abstract

In this paper we investigate two elements that would make stochastic simulations more realistic. The first is that we do not know the exact values of the parameters of a model which have been estimated from finite data, so it is appropriate to allow for this parameter uncertainty. We use the information matrix in the maximum likelihood estimation process to obtain a possible multivariate normal distribution for the parameter values. This may entail using transformed versions of the parameters. Secondly, since the empirical data on which the models for several of our variables are based may be fat-tailed, it is better to use a more appropriate distribution than the Normal to represent them. We use what we call conical distributions, including Normal, Laplace and Hyperbolic, both straight and Skew. This, however, requires re-estimating all the parameters of the model and their distribution, so the two elements are interconnected. We find that the first element has an effect at long durations, the second at short durations, and both are important. We use our model for retail prices to investigate these elements fully and to show the method for other variables.

We investigate two aspects of the Wilkie model, which would apply to any other model for economic scenario generation. These appear separate but they are interconnected. The first is making an allowance for parameter uncertainty. We can do this though what we call a "hypermodel" in which the values of the parameters to be used in each simulation are themselves treated as random variables, with some given multivariate distribution.

The second new feature is in allowing for distributions for the innovations other than normal, so that the fat-tailedness and sometimes the skewness of the empirical data is better represented. This appears at first sight to be independent of the parameter uncertainty problem. However, our estimation of parameter values has so far always been by maximum likelihood estimation assuming normal residuals, and we allow for parameter uncertainty from the resulting Fisher

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information matrix. If we allow for a different distribution for the residuals, we should allow for that in the maximum likelihood estimation, from which we may get different values of the estimated parameters or for their standard errors, or both.

This whole process is rather complex, so we break it into three separate steps. In the first we retain normally distributed residuals, and show how we build a hypermodel to allow for parameter uncertainty. In the second we keep the parameters as estimated from our normality assumption, but investigate the distribution of residuals. In the third we use the new distribution of residuals in the maximum likelihood estimation and see what difference it makes to the estimated values of the parameters of the "skeleton" - that part of the model that does not involve the residuals/innovations.

In this paper we restrict ourselves mainly to the model for retail prices, which is the simplest part of the Wilkie model, but also which underlies almost all the other parts. However, we also make reference to other aspects of the model where appropriate.

We have carried out 1,000,000 simulations, each for 500 years, for each of the chosen distributions. We can readily conclude that, in the short term, the choice of distribution is important, whereas in the long term the use of hypermodel is important. But to get more realistic simulations all round, both are necessary.

Keywords: Wilkie model; retail prices; parameter uncertainty; hypermodel; distribution of innovations.