

Dynamic Robust Return Risk Measures: Time-Consistency and g -Expectations

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Abstract

Robust return risk measures have been introduced by [1] mainly for two purposes. First, to emphasize the difference between risk measurement in terms of monetary values and in terms of returns. Second, to take into account ambiguity with respect to the probabilistic model P given a priori, by means of multiple priors [6], variational preferences [10], or homothetic preferences [4, 3, 9]. In particular, in [1] we defined, axiomatized and studied robustified versions of Orlicz premia and of Haezendonck-Goovaerts risk measures. See [7, 5, 8, 2] for the classical (non-robust) definitions.

In this paper we extend to a dynamic setting the notions above and we extensively analyze the properties of the resulting dynamic robust return risk measures. Furthermore, we provide characterization results of their time-consistency based on mixture representations, based on dual representations, and based on recursive construction via generators. In particular, we show that time-consistency of robust Orlicz premia and of Haezendonck-Goovaerts risk measures are strongly related.

We also establish a new link to g -expectations (defined in [11]) and BSDEs and, finally, we illustrate our results in a few examples and applications.

Keywords: Risk measures; Insurance premia; Time-consistency; Robustness; g -expectations.

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