Some properties of compound geometric distributions through the classical risk model

Georgios Psarrakos [1]

Department of Statistics and Insurance Science
University of Piraeus, Greece

Abstract

Let the claim sizes $X_1, X_2, \ldots$ be i.i.d. supported on $[0, \infty)$ with distribution function F, and N be the number of claims in the collective risk model. The aggregate claim amount is

$$S = \sum_{i=1}^{N} X_i, \quad N, X_i \text{ are independent}$$

with distribution function

$$F_S(x) = Pr(S \leq x) = \sum_{n=0}^{\infty} Pr(N = n)\, F^{*n}(x), \quad x \geq 0,$$

where F^{*n} denotes the n-fold convolution of F with itself. Usually, in stochastic risk models the random variable N is a geometric distribution of the form

$$Pr(N = n) = (1 - \phi)\, \phi^n, \quad \phi \in (0, 1), n = 0, 1, 2, \ldots,$$

and

$$E(S) = E(N)\, E(X_i) = \frac{\phi\, \mu_F}{1 - \phi}\,.$$

In many situations, the computation or the numerical estimation of $P(S \leq x)$ is not easy. In this talk, we give some characterization results through the classical risk model. In particular, we consider the distribution of the deficit at the time of ruin and we provide some distributional properties in the case where the claim size distribution has a heavy or light tailed distribution, such as Pareto or exponential claims. Examples are given to illustrate our results.

Keywords:
Classical risk model; Deficit at ruin; Ruin probability; Equilibrium distribution; Mean residual lifetime.

Acknowledgements: This work has been partly supported by the University of Piraeus Research Center.

[1]E-mail address: gpsarr@unipi.gr

References

[1] Albrecher, H. and Kortschak, D. (2009). On ruin probability and aggregate claim representations for Pareto claim size distributions. Insurance: Mathematics and Economics 45, 362–373.

[2] Gerber, H.U., Goovaerts, M.J. and Kaas, R. (1987). On the probability and severity of ruin. ASTIN Bulletin 17, 151–163.

[3] Ramsay, C.M. (2003). A solution to the ruin problem for Pareto distributions. Insurance: Mathematics and Economics 33, 109–116.