Statistical Inference for Discretely Observed Markov Processes, with Application to Credit Rating Transitions

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This talk presents two new methodolocial findings: first, a new analytical expression for the observed Fisher information matrix of discretely observed Markov processes. Second, an analytical expression for the Delta method for matrix exponential transformations of Markov generator matrices. The results can help to reduce computing time compared to existing approaches and increase interpretability of parameter estimates when applied in a credit risk context. The expressions are illustrated by Moody's corporate rating data.

Markov processes can be easily employed to model credit rating transitions if continuous-time rating migration data are available. However, such data are often very expensive whereas discrete-time data on an annual or semi-annual basis are available for free. Against this background, an important branch in the literature on Markov models in credit risk concentrates on the estimation of discretely-observed Markov processes.

Maximum likelihood estimates of the parameters of discretely observed Markov processes can not be derived analytically but e.g., by an expectation-maximization algorithm, see e.g., [1]. Moreover, an observed Fisher information matrix can be derived and e.g., employed to calculate confidence intervals or to conduct statistical tests.

An analytical expression of this observed Fisher information matrix has been first described by [3]. Their result is based on a general relationship between the Fisher information matrix and the expectation-maximization algorithm in latent variable problems, which has been found by [2] and was already partially concretized for discretely observed Markov processes by [1]. [3] however use two theoretical results: first an expression that partial derivatives of matrix functions inside a matrix exponential can be represented by a specific integral. Second, that these integrals can be represented by a single matrix exponential function.

However, by having these two results available, the indirection of deriving the Fisher information matrix by components of the expectation- maximization algorithm is no longer necessary. Instead, the information matrix can be directly obtained as the negative second order derivative of the discretely observed Markov process log likelihood function. The resulting expression is of distinctly lower complexity than the result of [3] so that the computing time compared to the expression of [3] will be reduced to less than one half.

The Fisher information matrix may be employed to quantify the stability of Markov generator matrices. However, on this level parameter estimates can only be seen as infinitesimal intensities

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and thus for credit ratings, lack interpretability. Instead, matrix exponential transformations of generator matrices yield discrete-time (rating) transition probability matrices and have an intuitive explanation. Against this background, an analytical expression for the Delta method of matrix exponential transformations of Markov generator matrices is derived.

Keywords: Markov Process, Fisher Information Matrix, Confidence Interval, Credit Rating Transitions

References

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