Robust Utility Maximization with Lévy Processes

Ariel Neufeld *[1] and Marcel Nutz †[2]

[1]Department of Mathematics, ETH Zurich.
[2]Department of Statistics, Columbia University, New York

Abstract

We present a robust utility maximization problem of the form

$$\sup_{\pi} \inf_{\mathbb{P}} \mathbb{E}^{\mathbb{P}}[U(W_T^{\pi})] \tag{1}$$

in a continuous-time financial market with jumps. Here W_T^{π} is the wealth at time T resulting from investing in d stocks according to the trading strategy π and U is either the logarithmic utility $U(x) = \log(x)$ or a power utility $U(x) = \frac{1}{p}x^p$ for some $p \in (-\infty, 0) \cup (0, 1)$. The infimum is taken over a class $\mathfrak{P}$ of possible models $\mathbb{P}$ for the dynamics of the log-price processes of the stocks. More precisely, the model uncertainty is parametrized by a set Θ of Lévy triplets (b, c, F) and then $\mathfrak{P}$ consists of all semimartingale laws $\mathbb{P}$ such that the associated differential characteristics $(b_t^{\mathbb{P}}, c_t^{\mathbb{P}}, F_t^{\mathbb{P}})$ take values in Θ, $\mathbb{P} \times dt$-a.e. In particular, $\mathfrak{P}$ includes all Lévy processes with triplet in Θ, but unless Θ is a singleton, $\mathfrak{P}$ will also contain many laws for which $(b_t^{\mathbb{P}}, c_t^{\mathbb{P}}, F_t^{\mathbb{P}})$ are time-dependent and random. Thus, our setup describes uncertainty about drift, volatility and jumps over a class of fairly general models.

Our first main result shows that an optimal trading strategy $\widehat{\pi}$ exists for (1). This strategy is of the constant-proportion type; that is, a constant fraction of the current wealth is invested in each stock. We compute this fraction in semi-closed form, so that the impact of model uncertainty can be readily read off. Thus, our specification of model uncertainty retains much of the tractability of the classical utility maximization problem for exponential Lévy processes. This is noteworthy for the power utility as $\mathfrak{P}$ contains models $\mathbb{P}$ that are not Lévy and in which the classical power utility investor is not myopic. Moreover, while the classical log utility investor is myopic in any given semimartingale model, this property generally fails in robust problems, due to the nonlinearity caused by the infimum–retaining the myopic feature is specific to the setup chosen here, and in particular the (nonlinear) i.i.d. property of the increments of the log-prices under the nonlinear expectation $\inf_{\mathbb{P}\in\mathfrak{P}} \mathbb{E}^{\mathbb{P}}[\,\cdot\,]$ in the sense of [1, 3].

Under a compactness condition on Θ, we also show the existence of a worst-case model $\widehat{\mathbb{P}} \in \mathfrak{P}$. This model is a Lévy law and the corresponding Lévy triplet $(\widehat{b}, \widehat{c}, \widehat{F})$ is computed in semi-closed form. More precisely, our second main result yields a saddle point $(\widehat{\mathbb{P}}, \widehat{\pi})$ for the problem (1) which may be seen as a two player zero-sum game. The strategy $\widehat{\pi}$ and the triplet $(\widehat{b}, \widehat{c}, \widehat{F})$ are

*E-mail address: ariel.neufeld@math.ethz.ch (Speaker at the conference)
†E-mail address: mnutz@columbia.edu

characterized as a saddle point of a deterministic function. The fact that $\widehat{\mathbb{P}}$ is a Lévy model may be compared with option pricing in the Uncertain Volatility Model, where in general the worst-case model is a non-Lévy law unless the option is convex or concave.

This talk is based on joint work in [4] with Marcel Nutz.

Keywords: Utility maximization; Knightian uncertainty; Nonlinear Lévy process.

References

[1] M. Hu and S. Peng. G-Lévy processes under sublinear expectations. *Preprint arXiv:0911.3533v1*, 2009.

[2] A. Neufeld and M. Nutz. Measurability of semimartingale characteristics with respect to the probability law. *Stochastic Process. Appl.*, 124(11):3819–3845, 2014.

[3] A. Neufeld and M. Nutz. Nonlinear Lévy processes and their characteristics. *Trans. Amer. Math. Soc.*, 369(1):69–95, 2017.

[4] A. Neufeld and M. Nutz. Robust Utility Maximization with Lévy Processes . *Mathematical Finance*, 28(1):82–105, 2018.