Multidimensional optimization of dividends with the possibility of irreversible change of regime: A convergent numerical scheme

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Abstract

In [1], it was considered the problem of maximizing the expectation of discounted dividends until a stopping time. This stopping time was defined as the minimum between the ruin and a decision time that stops the process (with a possible extra dividend payment at that time). The uncontrolled surplus was assumed to be a compound Poisson process with negative jumps. In the present work, we generalize this problem to a multidimensional setting (n+2 associated companies) with the possibility of an irreversible change of regime. For instance, a change of regime could be that the manager of the insurance company can decide to merge the companies at any time (see [2]). We also include in this work a penalty/reward function at ruin time. As in the one-dimensional case, it can be proved that the optimal value function V can be characterized as the smallest viscosity super-solution of the associated non-linear integro-differential Hamilton-Jacobi-Equation. In this case, the HJB equation is a maximum between n+2 operators equal to zero and the n+2 regions in which each operator applied to the optimal value function V is zero determine the optimal strategy. The hardest part of is to actually find these regions since there are not closed form solutions. The main contribution of this work is to approximate these regions and V by a convergent numerical procedure; we propose to consider a family of admissible strategies in a suitable grid that satisfy a discrete version of the HJB equation and show that the value function of these strategies converge locally uniformly to the optimal value function V as the mesh size goes to zero. We also show numerical examples.

Keywords: Mixed singular/switching control problem; multidimensional compound Poisson process; optimal dividends; optimal switching; Hamilton-Jacobi-Bellman equation; viscosity solutions; convergence of numerical scheme.

References

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