Risk measures based on benchmark loss distributions

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Abstract

We introduce a class of quantile-based risk measures that generalize Value at Risk (VaR) and, likewise Expected Shortfall (ES), take into account both the frequency and the severity of losses. Under VaR a single confidence level is assigned regardless of the size of potential losses. We allow for a range of confidence levels that depend on the loss magnitude. The key ingredient is a benchmark loss distribution (BLD), i.e. a function $\alpha : [0, \infty) \to [0, \infty)$ that associates to each potential loss a maximal acceptable probability of occurrence. The corresponding risk measure

$$\rho_\alpha(X) = \inf\{m \in \mathbb{R}\,;\ \mathbb{P}(X - m \leq u) \geq \alpha(u),\ u \in [0, \infty)\}$$

determines the minimal capital injection that is required to align the loss distribution of a risky position (X stands for a random variable on a given probability space representing losses at a pre-specified future point in time) to the target BLD. By design, one has full flexibility in the choice of the BLD profile and, therefore, in the range of relevant quantiles. Special attention is given to piecewise constant functions and to tail distributions of benchmark random losses, in which case the acceptability condition imposed by the BLD boils down to first-order stochastic dominance. Starting from the (computationally useful) equivalent formulation

$$\rho_\alpha(X) = \sup_{u \in [0,\infty)} \mathrm{VaR}_{\alpha(u)}(X) - u$$

we provide a comprehensive study of the main finance theoretical and statistical properties of ρ_α with a focus on its comparison with VaR and ES. In particular, we show that BLD-based risk measures are not coherent but are robust and, at the same time, capture tail risk in a much better way than VaR. Moreover, contrary to ES, they behave well in the presence of

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infinite-mean models. Our results are illustrated by applications to capital adequacy, portfolio risk management, and catastrophic risk.

Keywords: risk measures, loss distributions, tail risk, capital adequacy, portfolio management, catastrophic risk, robustness, backtesting

Acknowledgements: This research started while the second and third author were visiting the Department of Mathematics of the University of Milan and the Department of Statistics and Quantitative Methods of the University of Milano-Bicocca supported by the ACRI Research Prize 2017. The second author acknowledges financial support from the ETH Foundation. The authors would like to thank Pablo Koch-Medina, Andreas Tsanakas, and Ruodu Wang for useful discussions and suggestions on an earlier version of this paper.