Optimal Asset Allocation in Pension Funds Under Consideration of Higher Moments

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Abstract

In the current low interest rate environment, a proper asset allocation strategy is crucial for financial institutions like life insurers and pension funds. Often being obliged to grant their clients a guaranteed interest rate on their savings, they face the issue of having to find a suitable combination of asset classes to invest in. With respect to Swiss pension funds, this topic is of great importance. As funds face large obligations towards their policyholders, it is necessary for them to have an investment strategy that balances expected return and volatility. While the expected return must suffice to earn the promised interest for the clients, the volatility should not be too high. Otherwise, years with very low capital market returns could lead to a strong decrease of the funding ratio and thus put the fund's solvency at risk. It is therefore crucial to choose a combination of assets that meets the needed returns while maintaining a certain safety level. In our work, we seek to examine this subject by looking at specific asset allocation techniques under selected return distribution assumptions. To this respect, we study the impact that higher moments have when included in the decision taker's utility function. We analyze how using different distributions for simulating the assets leads to a better fit of the historic data and thus to improved simulation results. In order to compare the resulting allocations, we simulate the assets of a fund and analyze the results after one period.

To start our study, we consider a pension fund with a given asset-liability situation and which is regulated by prevalent rules of the Swiss pension fund system. Our aim is to find the asset allocation that allows the fund to reach a given expected target return, a given funding ratio or a given low underfunding probability. Having established an optimal allocation strategy, we simulate the assets of the fund. For this, we first make use of a multivariate normal distribution. Figure 1 displays the asset allocations in the efficient portfolios. For every combination of target returns and corresponding minimum variances, it depicts the optimal shares $\boldsymbol{\alpha}^*$ of the five asset classes. It can be seen that for low target returns the efficient portfolio mainly consists of money market investments. As the drift μ increases, this share decreases while the shares of the remaining assets increase, with the government bonds having the second-highest percentage. As μ grows further, the share of the hedge funds and the government bonds evolve similarly at first. The changes in the asset shares are linear up to a drift of almost 3.5%, where the hedge funds reach their maximum share of 15%. Subsequently, the share of the money market begins

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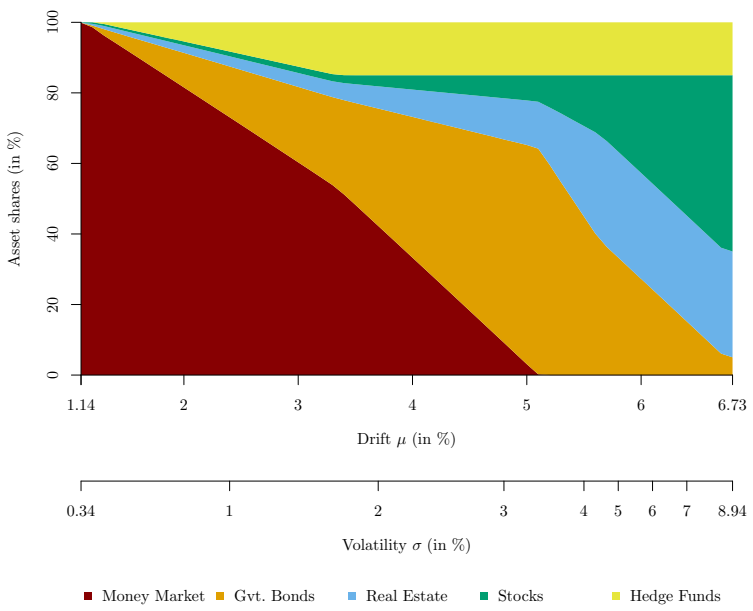

Figure 1: Optimal asset allocations for target return μ and normally distributed returns.

to decrease steeply and the portfolio weights shift more towards the government bonds, the real estate and the stocks. This effect enhances further when the share of the money market drops to zero. The changes appear as kinks in the course of the asset shares. For the government bonds a decrease takes place for high returns. For the portfolio with the highest return, which is located on the right end of the graph, the hedge funds, the stocks and the real estate achieve their investment limits.

Expectedly, our results indicate that the use of an extended utility function does indeed lead to a shift in the optimal portfolio weights. Consequently, main portfolio characteristics change. Among others, a strong increase of the volatility takes place. We are therefore able to say that using the minimum variance portfolio can cause misleading security. Working with an extended utility function that departs from the minimum variance framework and incorporates higher moments of returns consequently allows companies to asses their risk taking more adequately. Having analyzed the effects of an extended utility function and different asset distributions, we examine the impact of risk aversion. To this end, we look at a vector of risk aversion coefficients with respect to the second, third and fourth moment of the portfolio return. The results show that the outcome changes only little when varying the parameter related to the volatility. In contrast to this, we see strong variations when changing the coefficients of the skewness and kurtosis. As a consequence of this, the optimal portfolio weights shift together with its characteristics. It is thus necessary to use a more complex risk aversion when optimizing utility functions.

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