A Bivariate Model for the Limit-Order-Book: An Application of Hawkes Processes to Describe Dependence Structures in High-Frequency Finance

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Abstract

Large banks, sizable brokerage firms, market-makers and large investment funds face a very specialized challenge when seeking to trade stocks at high frequencies on behalf of their costumers or participants. Indeed, there is a need to determine optimal strategies that would allow to minimize losses and even create profit by efficiently executing pair trades throughout a given day. Current conditions make them highly exposed to market liquidity costs and micro-structure behavior. There is a need for bivariate models to evaluate the market impact and performance of high-frequency pair strategies.

This risk exposure comes as a result of the operational mechanism of Today's markets. As it turns out, the evolution of prices in modern markets are the result of interactions of different types of buy and sell orders through a double auction mechanism that stores and matches orders to facilitate trading. This order-flow is stored in a limit order book (LOB) that contains a list of buy and sell limit orders with their corresponding price and size at any given nanosecond. The so-called LOB is a snapshot of the liquidity available in the market at a given nanosecond and it evolves at very high frequencies in modern markets (approximately 60 book updates per second). Now, this mechanism produces a dependent liquidity risk when executing large pairs trades since the cost of a given trading strategy will be impacted by the liquidity available in two dependent LOB at the time of transaction as well as by the size of the order. As it turns out, large orders will be executed and matched to existing liquidity in each LOB producing an increasingly disadvantageous trading price since it will dig deeper and deeper into the LOB.

A key role in the risk analysis of such exposure can be played by an efficient market simulator model capable of reproducing stylized features of two dependent LOB's so that strategies can be tested and compared in terms of their market impact. There is a

fair amount of scientific literature dealing with the problem of devising and improving on simulating engines capable of reacting to large orders in the same way that the real market would do (see for instance [4] and [3]). Such models are able to reproduce key features of the LOB that are now known to be distinct price signatures such as spread, volumes, book slope and volatility. All of these variables provide a distinct signature for a given stock and its liquidity supply profile. In contrast, little exists in terms of modeling dependence between two LOB. A first step towards building a risk management tool to evaluate market impact in pairs trading is to calibrate the model parameters as to reproduce the signature features of two given stocks hence producing LOB profiles that are qualitative and quantitative similar to those observed during the day.

In this talk we present one such bivariate LOB simulator model that generalizes [1] and [2]. We use a multivariate self-exciting point process to model the arrival of LOB events. The so-called Hawkes process has been used in the literature to model micro-structural behavior but rarely in a bivariate context [5]. Using LOB data, we explore the suitability of this bivariate model to reproduce the high-frequency signature of two given stocks.

References

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