

A Factorization of a Lévy Process over a Phase-Type Horizon

Søren Asmussen and Jevgenijs Ivanovs*

Aarhus University

Abstract

Motivated by some pricing problems in equity-linked products [1], we consider the joint distribution of the supremum and the terminal value of a Lévy process over a time horizon τ that is the remaining lifetime of the insured, which by denseness may be appropriately modelled by a phase-type distribution [2, 3]. This joint law is derived from a new factorization identity for a Lévy process, which extends the classical Wiener-Hopf factorization from an exponential to a phase-type time horizon. One of the factors is defined using time reversal of the phase process. It is shown that there are a variety of time-reversed representations, all yielding the same factor. Consequences of this are discussed and examples provided. Additionally, some explicit formulas for the joint law of the supremum and the terminal value of the process at τ are given.

Keywords: equity-linked benefits, Wiener-Hopf factorization, splitting, time reversal, phase-type distribution.

References

- [1] H.U. Gerber, E.S.W. Shiu & H. Yang (2013), “Valuing equity-linked death benefits in jump diffusion models.” *Insurance: Mathematics and Economics* **53**, 615–623.
- [2] X.S. Lin & X. Liu (2007), “Markov aging process and phase-type law of mortality.” *North American Actuarial Journal* **11**, 92–109.
- [3] A.H. Zadeh, B.L. Jones & D.A. Stanford (2014), “The use of phase-type models for disability insurance calculations.” *Scandinavian Actuarial Journal* **8**, 714–728.

*E-mail address: jevgenijs.ivanovs@math.au.dk