

# Using Graduation to Modify the Estimation of Lee-Carter Model for Small Populations

Jack C. Yue <sup>\*1</sup>, Hsin-Chung Wang <sup>†2</sup>, and Tzu-Yu Wang <sup>‡3</sup>

<sup>1</sup>Department of Statistics, National Chengchi University, Taipei, Taiwan, Republic of China

<sup>2</sup>Department of Finance and Actuarial Science, Aletheia University, New Taipei City, Taiwan, Republic of China

<sup>3</sup>Department of Mathematical Sciences, National Chengchi University, Taipei, Taiwan, Republic of China

## Abstract

The study of human longevity has been a popular research topic due to the prolonging of life. However, the limited availability and poor quality of elderly data increase the difficulty of mortality modeling. It is particularly challenging if the size of the target population is small, and the parameter estimation of stochastic mortality models can be distorted. For example, the estimates of parameters in the Lee-Carter model can be biased when the population size is 500,000 or less. In this study, we aim to provide a possible solution to deal with the problem of biased estimates for the Lee-Carter model in the case of small populations.

We propose graduation methods to modify the parameters' estimates of mortality models, similar to the process of constructing life tables where mortality rates are smoothed to remove the irregularity of some observed values. The graduation methods, including Whittaker graduation and partial standard mortality ratio [10], are applied to the mortality rates to smooth the parameters' estimates of Lee-Carter model, and the results are compared to those of coherent Lee-Carter model [11]. We use computer simulation to evaluate the proposed approach and it does have smaller fitting errors when the population size is small.

**Keywords:** Small area estimation, Standard mortality ratio, Graduation, Lee-Carter model, Longevity risk

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\*E-mail address: [csyue@nccu.edu.tw](mailto:csyue@nccu.edu.tw)

†E-mail address: [au4369@mail.au.edu.tw](mailto:au4369@mail.au.edu.tw)

‡E-mail address: [102701023@nccu.edu.tw](mailto:102701023@nccu.edu.tw)

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