Mortality Risk Modelling with Renewal Process

Selin Değirmenci *[1] and Şule Şahin †[2]

[1]Karabuk University, Department of Actuarial Sciences and Risk Management, Karabuk, TURKEY

[2]University of Liverpool, Institute For Financial and Actuarial Mathematics, Liverpool, UK

Abstract

Insurance companies are exposed to the risk which can affect their long term liabilities, such as interest rate risk and mortality risk. Mortality risk arises from a shorter life time than anticipated of an individual or group. When a catastrophic event occurs, mortality rates are higher than expected and life insurers have to make sudden payouts to many policyholders. This causes great financial implications and affect the financial strenght of the life-insurance and pension companies. Thus management of mortality risk is fundamental for life insurance and pension industries.

The number of catastrophic events has risen in the last four decades. According to The World Disasters Report in 2016, rising global temperatures causes the global climate change and rising natural disasters. These climate change and natural disasters cause catastrophic events. Global climate change bring fluctuations in wheather patterns, mega-storms, wild summer and winter conditions, floods and other disasters. For instance, people in India, Pakistan, France, Egypt and Italy are influenced by the extreme heats and about 5000 people died because of this extreme heat. Natural disasters have come off more frequently for the last 20 years according to the statistics of natural disasters. In the 1970s there were roughly 100 catastrophic events per year. This number has more than tripled in the last decade [1]. This catastrophic events causes huge jumps in mortality rates. To manage this mortality risks we must model the mortality with mortality jump factors. Considering the increasing number of catastrophic events, we must include the history of events into our mortality model.

In this paper we model mortality by using the Lee-Carter model with mortality jumps. We proposed a new approach to model mortality index by using Merton model to reflect jump effects [2]. In Merton model jump severities are distriuted as normal. But the mortality index show leptokurtic features and to reflect leptokurtic features of the mortality index we assume that the jump severities are distributed as exponential. As we mentioned before, the history of events can give us the information about the repetition frequency of jumps, we must include the history of events into the modelling process. In poisson process, waiting times between the events are independent and exponentially distributed. Since the memoryless property of

*E-mail address: selindegirmenci@karabuk.edu.tr

†E-mail address: Sule.Sahin@liverpool.ac.uk

exponential distribution, an arrival time is independent from the last arrival. This assumption is not realistic and the history of process can give information about future arrivals. For this purpose we model jump counts by using renewal process. Since the inter-arrival times of this process are i.i.d. random variables we need to select the inter-arrival times's distribution. We use Weibull distribution because of its non-constant hazard function property. According to our model the mortality index is as follows[1]:

$$dk_t = \mu dt + \sigma W_t + d\sum_{i=1}^{N(t)} (V_i - 1), \tag{1}$$

where W_t is a standard Brownian motion, $N(t)$ is renewal process. V_i is a squence of independent identically distributed nonnegative random variables and $Y = log(V)$ has an exponential distribution with the density:

$$f_Y(y) = \eta e^{-\eta y}, \quad \eta > 0 \tag{2}$$

Our aim is to find a more realistic mortality model and capture the effects of catastrophic events, so that an insurance company or a pension plan could meet its financial liabilities and hedge its mortality risks.
By using the Weibull renewal process as counting process, we can handle both overdispersed and underdispersed data which are likely to be seen in practice. Although the calculations of this process has difficulties, we found that the history of catastrophic events has an important role in mortality modelling. This model can capture the shape, the trend and the jumps of the mortality time series. Because of these properties, this model can be seen as a more suitable model for the mortality modelling.

Keywords: Mortality risk, jump diffusion process, renewal process.

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