Optimal Investment to Minimize the Expected Cumulative Drawdown

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Abstract

Drawdown measures an investor's sustained loss from a historic high-water mark (maximum). In fund management industry, drawdown is one of the most frequently quoted indices for downside risks, for instance, in performance measures such as the Calmar ratio and the Sterling ratio. Controlling the risk of drawdown is crucial as a significant drawdown or a long period in drawdown (i.e., a long period below historic high-water mark) may trigger massive fund redemption. This paper focuses on investigating the optimal trading strategies when the objective is to dynamically minimize drawdown-based risk metrics. Compared to the prevailing risk measures that are based on terminal positions, for instance, Value at Risk and Conditional Tail Expectation, a drawdown-based risk metric introduces an addtional dimension along the paths, i.e., running maximum, and thus is more informative in depicting the downside risk profile of a fund. In particular, the expected cumulative drawdown is considered, which contains information on both magnitude and temporal dimension of drawdown risk. Under the Black-Scholes framework, closed-form optimal trading strategies, expressed in terms of confluent hypergeometric functions, are derived by utilizing Legendre transform on the associated Hamilton-Jacobi-Bellman (HJB) equation. The results show that investors tends to become more conservative as drawdown level decreases.

Keywords: Portfolio optimization; Drawdown risk; Dual approach; Confluent hypergeometric function.

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