Capital allocations for classical and set-valued risk measures

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Abstract

To address future uncertainty about their net worth, firms, insurances and in general portfolio managers are often imposed to hold a so called risk capital, that is, an amount of riskless assets in order to hedge themselves. This fact then raises the issue of how to share all this immobilized capital in an a priori fair way among the different lines or business units (see, for example [4], [6]).
As risk capital is commonly accepted in the literature to be modeled through the use of risk measures [1], [4], [5], [10]), capital allocation problems in risk management and the theory of risk measures are naturally linked.
Starting from Deprez and Gerber's ([7]) work on convex risk premiums, Tsanakas ([11]) defines a Capital Allocation Rule (C.A.R) for Gateaux differentiable risk measures inspired to the game theoretic concept of Aumann and Shapley value ([2]), and studies its properties for some widely used classes of convex risk measures, also providing explicit formulas. His analysis leaves anyway substantially open the case of general non Gateaux-differentiable risk measures (although he treats the case of distortion exponential risk measures, but it is easy to find other meaningful examples of convex and quasi-convex non Gateaux differentiable risk measures) as well as the study of quasi-convex risk measures, whose importance is well recognized in the literature ([3], [8]).
In the present work we propose a family of C.A.R. for (convex and quasi-convex) real-valued risk measures based on the dual representation theorems and on subdifferentials, study their

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properties and show that they reduce to Tsanakas' one, when we assume Gateaux differentiability. In the meantime, we discuss the suitability of the use of quasi-convex risk measures for capital allocation purposes. We then focus on capital allocations for set-valued or vector-valued risk measures and premia, by extending the C.A.R. above to this more general framework.

Keywords: Capital allocation rules, Convex and quasi-convex risk measures, Aumann-Shapley value, Gateaux differential, Subdifferentials.

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