Procyclicality of Empirical Measurements of Risk in Financial Markets

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Abstract

In this study we consider the risk estimation as a stochastic process based on the Sample Quantile Process (SQP) - which is a generalization of the Value at Risk (VaR) calculated on a rolling sample. Using SQP's, we are able to show and quantify the pro-cyclicality of the current way financial institutions measure their risk. Analysing eleven stock indices, we show that, if the past volatility is low, the historical computation of the risk measure underestimates the future risk, while in periods of high volatility, the risk measure overestimates the risk.

Moreover, using a simple GARCH(1,1) model, we conclude that this pro-cyclical effect is related to the clustering of volatility. At the same time, we prove that part of the procyclicality is intrinsically caused by the way of the historical risk estimation. We argue that this has important consequences for the regulation in times of crisis.

Details

There is an accepted idea that risk measurements are pro-cyclical. Still, there have been little attempts to study emperically the direct relation between risk measure estimates and procyclicality, and even less to quantify it.

The Sample Quantile Process. Our approach requires a dynamic reformulation of the risk measurement. For this purpose we use the sample quantile process (SQP) first introduced in [3] and developed in [1, 2] as a way to measure risk: The Sample Quantile Process $(Q_{\mu,\alpha,T,t}(L))_{t\geq 0}$ of a loss stochastic process $L=(L_t)_t$ at threshold α, with respect to a random measure μ defined on $\mathbb{R}^+$, is defined by

$$Q_{\mu,\alpha,T,t}(L) = \inf\left\{ x : \frac{1}{\int_{t-T}^{t}\mu(s)ds}\int_{t-T}^{t}\mathbf{1}_{(L_s\leq x)}\mu(s)ds \geq \alpha \right\}. \quad (1)$$

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The SQP is clearly a dynamic generalization of the VaR. Indeed, choosing μ in (1) as the Lebesgue measure, corresponds to a rolling window VaR (denoted by $Q_{0,\alpha,T,t}(L)$).

Measuring procyclicality. Using the SQP with different choices of its random measure μ, we empirically explore its dynamic behavior and its ability of predicting correctly the future risk. We measure the latter with the ratio of SQPs denoted by $R_{\mu,\alpha,T}(t)$:

$$R_{\mu,\alpha,T}(t) = \frac{Q_{0,\alpha,1}(t+1y)}{Q_{\mu,\alpha,T}(t)}. \quad (2)$$

It quantifies the difference between the historically predicted risk $Q_{\mu,\alpha,T}(t)$ and the estimated realized future risk $Q_{0,\alpha,1}(t+1y)$ (considered a posteriori at the time t plus one year later). Since the volatility is high in crisis time and low when there is little information hitting the market, it can be used as a proxy for the market state. Thus, we perform an analysis of the SQP ratio conditioned to the realized volatility, as done in [4] for the autocorrelation of the log-returns. This way we link the quality of future risk estimation with the question of procyclicality. We find that the realized SQP ratios are negatively correlated with the annualized volatility.

Explanations. We run Monte Carlo simulations using different models (constant-volatility, GARCH, long-memory processes) and find that a GARCH(1,1) presents a very similar behavior as the data. Also, we observe that a constant volatility model exhibits some negative correlation, too. Thus, we cannot attribute the negative dependence solely to the features of the GARCH. Additionally, we compute the dependence values theoretically. We look at the correlation between the SQP-ratio at time t, $R_{\mu,\alpha,T}(t)$, and our indicator of the market state at time t, the standard deviation σ_t, (as well as of their corresponding sample estimates, denoted with a hat):

$$cor(R_{\mu,\alpha,T}, \sigma_t) \text{ and } cor(\hat{R}_{\mu,\alpha,T}, \hat{\sigma}_t) \quad (3)$$

The theoretical values confirm our observations from the Monte Carlo simulations. Recalling the negative dependence in the constant-volatility (iid) model, it implies that part of the negative dependence is intrinsically caused by the way the risk is estimated, using a historical VaR!

Keywords: risk measure; sample quantile process; stochastic model; VaR; volatility.

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