# Incorporating taxation in the valuation of variable annuity contracts: the case of the guaranteed minimum accumulation benefit

Jennifer Alonso-García [*1,2], Michael Sherris[†2,1], Samuel Thirurajah [‡2,1], Andres Villegas [§2,1], and Jonathan Ziveyi [¶2,1]

[1]ARC Centre of Excellence in Population Ageing Research (CEPAR), UNSW Sydney, Australia.
[2]School of Risk and Actuarial Studies, UNSW Sydney, UNSW Business School, Australia.

## Abstract

Variable annuities (VA) offer an opportunity to participate in the equities market while ensuring minimum guarantee levels in case of poor market performance. We study guaranteed minimum accumulation benefits (GMABs) which are a subclass of GMLBs guaranteeing the return of the premium paid by the policyholder, or a a higher stepped up value at the end of the accumulation period of the contract. GMAB riders have gained a lot of interest in the actuarial literature [1, 2, 3]. In this paper, we provide a partial differential equation approach to pricing and computing fair fees for a GMAB contract which can be surrendered anytime prior to maturity.

One of the main attractive features of VAs is the opportunity that they provide for tax-advantaged investing [4]. More recently, researchers have established the importance of incorporating *taxation* into the valuation of guaranteed minimum withdrawal benefit (GMWB) products as it yields fees consistent with empirical observations [5], and explains industry practice of including a free death benefit guarantee in VAs [6]. We consider both the Australian and US tax systems in our valuation framework. In the US, capital losses can be offset by capital gains, whereas in Australia it is not possible, among other differences. We examine the impact of *tax* on a policyholder that behaves rationally with respect to the post-tax value of the contract.

We formulate the valuation problem of a GMAB contract from the policyholder perspective as a free boundary problem which can be solved using the method of lines. The method of lines is a fast, accurate and computationally efficient algorithm for solving free boundary problems. In

[*]E-mail address: j.alonsogarcia@unsw.edu.au
[†]E-mail address: m.sherris@unsw.edu.au
[‡]E-mail address: samuel.thirurajah@unsw.edu.au
[§]E-mail address: a.villegas@unsw.edu.au
[¶]E-mail address: j.ziveyi@unsw.edu.au

computing how policyholder surrender behaviour affects the insurer's total liabilities, we have also considered the difference between fee rates which are fair from the demand and supply side. The numerical results illustrate that the risk-free rate, volatility and surrender rate have strong influence on optimal surrender. This impact is contingent on how much fees the policyholder has paid up to the exercise decision point and also on the particular rules of the tax system, such as whether or not capital losses are permitted.

The presence of taxation drives a wedge between policyholder and insurer valuations. We find that, when capital losses cannot be offset by capital gains, the separation between the valuation curves increases as tax increases, reflective of the increasing value of the contract to the governments. However, the insurer liabilities are more robust to the tax rate compared to the contract value to the policyholder. In other words, the government obtains most of their revenue from the policyholder rather than the insurer in the imposition of the tax system. If capital losses can be offset by capital gains, policyholder's optimal surrender behavior changes to the benefit of the insurer (all else equal), reducing the insurer liabilities for any fee rate they choose to charge. Indeed, the policyholder is only likely to surrender after having paid a lot of fees and incurring losses they can offset. Of course, this is done at the expense of the government who bears the losses in this case.

**Keywords:** Variable annuity, GMAB/GMMB, Method of lines, taxes, pricing

**Acknowledgements:** We gratefully acknowledge funding from "UNSW Business School - Linkage Research Seed Funds - Demystifying Variable Annuities Contracts - Empirical Analysis".

# References

[1] Bauer, D., A. Kling, and J. Russ (2008). "A universal pricing framework for guaranteed minimum benefits in variable annuities." *ASTIN Bulletin: The Journal of the IAA* 38 (2), 621-651.

[2] Bernard, C., A. MacKay, and M. Muehlbeyer (2014). "Optimal surrender policy for variable annuity guarantees." *Insurance: Mathematics and Economics* 55, 116-128.

[3] Kang, B. and J. Ziveyi (2018). "Optimal surrender of guaranteed minimum maturity benefits under stochastic volatility and interest rates." *Insurance: Mathematics and Economics*. Available online.

[4] Milevsky, M. A. and K. Panyagometh (2001). "Variable annuities versus mutual funds: a monte-carlo analysis of the options." *Financial Services Review* 10 (1-4), 145-161.

[5] Moenig, T. and D. Bauer (2015). "Revisiting the risk-neutral approach to optimal policyholder behavior: A study of withdrawal guarantees in variable annuities." *Review of Finance* 20 (2), 759-794.

[6] Moenig, T. and D. Bauer (2017). "Negative marginal option values: The interaction of frictions and option exercise in variable annuities." *Working Paper*