Stochastic Model for Erythropoiesis

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Abstract

Erythropoiesis is a multistep process in which generations of diverse red blood cells are coupled with movement of cells through different stages of maturation (types). This process is based on amplification motors successively involved in the proliferation (and differentiation) of low number of stem cells to large numbers of mature cells.

In order to realize such an amplification, cells can make different type of division :

- renewal division, one cell gives rise to two daughter cells of the same type than her mother
- differentiation division, one cell gives rise to two daughter cells of the next stage of maturation (type) than her mother. More precisely, one cell of type i give rise to two cells of type $i+1$
- asymmetric division, one cell gives rise to two daughter cells with different type, one of the same type than her mother and an other more differentiated (of the next stage of maturation).

The type (stage of maturation) of a cell will be characterized by its dynamics, i.e. its division rate and its differentiation, renewal and asymmetric probabilities at a division event. A macroscopic point of view highlights one specific parameter by type which seems to play an important role in the understanding of amplification mechanism (called amplification factors in the following).

We will introduce a Stochastic Branching Pure Jump Process of three dimension modeling evolution in time of three different types of cells numbers (stem cells, progenitors and mature cells). Using a scale parameter K contributing in the initial condition of the first component and in the probabilities of each division happened, and averaging method (Kurtz) we will study convergence of our Multi-types Markovian Process when the parameter K tends to infinity. We will show that each component of this process has its own typical time scale closely related to the amplification factors. Our aim will be to show that the specifics parameters induce large fluctuations (that can play an unexpected and major role).